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Paper

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Abstract

The focus is on cross-sectional dependence in panel trade flow models. We propose alternative specifications for modeling time invariant factors such as socio-cultural indicator variables, e.g., common language and currency. These are typically treated as a source of heterogeneity eliminated using fixed effects transformations, but we find evidence of cross-sectional dependence after eliminating country-specific effects. These findings suggest use of alternative simultaneous dependence model specifications that accommodate cross-sectional dependence, which we set forth along with Bayesian estimation methods. Ignoring cross-sectional dependence implies biased estimates from panel trade flow models that rely on fixed effects.

KEYWORDS: Bayesian, MCMC estimation, socio-cultural distance, origin-destination flows, treatment of time invariant variables, panel models.
JEL: C18, C51, R11

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1 Introduction

The empirical trade literature has largely ignored the issue of cross-sectional dependence between countries in econometric estimation of empirical trade flow models (Baltagi, Egger and Pfaffermayr, 2014). In a cross-sectional setting, trade costs are incorporated using geographical distance between origin and destination dyads involved in trade flows, as well as socio-cultural factors. These might include: common language and currency, historical colonial relationships, common borders, trade agreements, etc. The latter are perceived as representing a generalization of distance that also influence trade costs. For example, common language and common currency should reduce trade costs.

In a panel data model setting, distance as well as socio-cultural factors (which we label generalized distance variables) are generally time invariant, so they are modeled using fixed effects. In a conventional panel setting the impact of time invariant variables reflects a source of heterogeneity, and introduction of appropriate fixed effects transformations are used to control for differences in the level of flows attributable to these country-specific time invariant factors.

This paper argues that generalized distance variables can be viewed as transmission channels and modeled as a source of cross-sectional dependence, frequently observed in trade flows (see Porojan, 2001). The objective is to introduce alternative simultaneous dependence specifications for modeling time invariant factors such as generalized distance variables. These model specifications accommodate cross-sectional dependence, which we set forth along with Bayesian estimation methods. Ignoring cross-sectional dependence implies biased estimates from panel trade flow models that rely on fixed effects.

The idea of our modeling approach becomes most clear for the case of a dummy variable reflecting common borders that is often introduced as a generalized distance variable that impacts trade costs. When introduced as an indicator variable, the implication is that higher levels of flows exist between countries with common borders, a heterogeneity effect. As an alternative treatment, common borders could be introduced as a first-order contiguity spatial weight matrix. A first-order contiguity spatial weight matrix, say W_b, for exports from a sample of N countries would be of dimension $N \times N$ with non-zero elements in the (i, j)th position if countries i and j share a common border, and zeros on the main diagonal. Multiplying the $N \times N$ spatial weight matrix with an $N \times 1$ vector of export/import flows f, or vector of income X produces a linear combination of *neighboring country* export/import flows $W_b f$, or income $W_b X$. Of course, we can take the same approach to forming an $N \times N$ matrix (say) W_c having non-zero elements in the (i, j)th position if countries i and j share a common currency or language, or exhibit colonial ties, etc. We will have more to say about this later, but we note that a vector $W_c X$ in this context represents a linear combination of income from countries showing a socio-cultural similarity measured in terms of common currency, language, colonial ties, and so on.

The vector $W_c f$ can be used to specify a model of cross-sectional dependence that reflects interaction between neighboring countries, neighbors to the neighboring countries etc., which result in global spillover impacts. The vector $W_c X$ captures contextual effects arising from neighboring countries, which result in local spillover impacts. A model that includes both $W_c f$ and $W_c X$ has been labeled a spatial Durbin model (SDM) specification in the spatial econometrics literature.

Of course, it is possible that trade flows reflect both a heterogeneity impact from time invariant fixed effects as well as global and local impacts of the type set forth above. We can test our alternative cross-sectional dependence specification for consistency with sample data on trade flows by eliminating fixed effects (through a transformation) and testing the transformed model for: cross-sectional dependence, contextual effects, or a combination of these. It is worth noting that our SDM specification allows for the presence/absence of cross-sectional dependence, and/or contextual effects as well as a combination of these. Using data transformed to eliminate time invariant fixed effects, we estimate a Bayesian panel SDM model to determine if cross-sectional dependence, contextual effects, or a specification with both of these is most consistent with a panel of imports and exports from a sample of 74 countries over the 38 year period from 1963 to 2000. Specifically, we consider 148 different panel data models, 74 models for imports of each country from all other 73 countries over the 38 year period in our sample, and another set of 74 panel data models for exports from each country to all other 73 countries, covering the 38 year time period.

Another methodological innovation is use of convex combinations of cross-sectional dependence weight matrix structures (see Pace and LeSage, 2002; Hazir, LeSage and Autant-Bernard, 2014; Debarsy and LeSage, 2017, 2018). The weight matrix structures are constructed to reflect:

spatial proximity between countries, as well as numerous types of socio-cultural proximity such as common currency, language, colonial ties, and so on. A convex combination of these multiple weight matrices (with associated parameters) is used to form a single weight matrix, where the parameters assign relative importance to each type of cross-sectional dependence. This approach allows us to treat socio-cultural factors (for example, common currency, common language, historical colonial relationships, trade agreements, and so on) that have been traditionally modeled as time invariant fixed effects as sources of cross-sectional dependence. Constructing weight matrices from indicator variables reflecting socio-cultural factors allows our SDM specification to model time invariant factors as network links between countries rather than simply a source of heterogeneity. We set forth Bayesian MCMC estimation methods for our model specification that allows for cross-sectional dependence reflecting interaction and global spillover impacts as well as contextual effects arising from neighboring countries.

Our Bayesian estimation approach allows for estimation and posterior inference on a vector of parameters that determines the relative importance of each type of cross-sectional dependence. Estimates are based on data transformed using an approach from Lee and Yu (2010) that eliminates both time-specific and country-specific fixed effects using an orthogonality transformation. If the generalized distance variables reflect only time invariant fixed effects, our model estimates should indicate no cross-sectional dependence or contextual effects. If this is not the case, we have evidence that these generalized distance variables have a greater impact on trade flows than the conventional heterogeneity view suggests.

Section 2 introduces conventional cross-sectional gravity models as used in the empirical trade literature, along with the notion of cross-sectional dependence. Section 3 discusses the formation of convex combinations of spatial and a host of socio-cultural proximity structures, and these are discussed in the context of the panel cross-sectional dependence specifications.

Section 4 outlines computationally efficient expressions for the static panel variant of the spatial Durbin model that we wish to estimate. Bayesian MCMC estimation and inference for the model specifications is discussed in section 5. Focus is on inference regarding the scalar parameters that determine the relative influence of five types of proximity that we consider (spatial, common language, common currency, trade agreements, and colonial ties) in our cross-sectional dependence specification. Debarsy and LeSage (2017) point to three computational challenges that arise for this type of model where the weight matrix W_c is a function of estimated parameters γ_ℓ ($\ell = 1, ..., L = 5$) indicating the relative importance assigned to each type of connectivity structure. Each of these is discussed in Section 6 along with approaches set forth in Debarsy and LeSage (2017) for overcoming these challenges. Given the mixture of multiple proximity channels of transmission, interpretation of the estimates from our specification differs from that in conventional spatial models. Section 6 discusses interpretation of estimates from the cross-sectional model specification.

Section 7 applies the approach to panel data on trade flows covering the 38 years from 1963 to 2000. We provide empirical estimates for the scalar parameters reflecting the mixture of spatial and socio-cultural measures of proximity, and test our cross-sectional model specification for consistency with the sample data. The magnitude of bias arising from cross-sectional dependence is assessed by examining estimates of local and global spillover effects, since these are restricted to zero in conventional panel trade models.

Section 8 provides conclusions. Appendix A presents information on data used as well as sources.

2 Empirical cross-section trade models

Most trade models specify aggregate bilateral demand equations of consumers in countries $j = 1, \ldots, N$ from producers in countries $i = 1, \ldots, N$ in the general form:[1]

$$f_{ijt} \quad = \quad l_{it} \; m_{jt} \; c_{ijt}^{\tau} \tag{1}$$

where f_{ijt} are bilateral exports of country i to country j at time t, l_{it} are exporter time-specific factors, m_{jt} are importer time-specific factors, while c_{ijt} is a measure of all bilateral trade costs from i to j at time t, with τ reflecting the partial elasticity of trade flows with respect to trade costs (see Baltagi, Egger and Pfaffermayr, 2014).

Specifics regarding what l_{it} and m_{jt} represent depend on the particular trade model. For example, using the model from Anderson and van Wincoop (2003) with a single sector, τ reflects a measure of the elasticity of substitution between products from different countries, and l_{it}, m_{jt}

[1]We deal only with the case where the number of importing and exporting countries is the same.

correspond to size measures such as gross domestic product (which we denote using X). This occurs since aggregated trade flows $F_{it} = \sum_{j=1}^{N} f_{ijt}$ represent total sales in country i at time period t which corresponds to gross domestic product. Finally, trade flows f_{ijt} are assumed to be inversely related to the bilateral trade costs c_{ijt}.

For the cross-sectional case where we have a single year, the model in Eq. (1) is double-indexed, resulting in a balanced panel in our case where the number of importing and exporting countries is equal. Applying a log-transformation to the deterministic part $h_{ij} = ln(l_i, m_j, c_{ij}^{\tau})$ of the model in Eq. (1) results in:

$$f_{ij} \quad = \quad \exp(h_{ij}\delta + \alpha + e_{ij}) \tag{2}$$

where we have added a log-additive disturbance term e_{ij} as well as an intercept term α, while δ is a conformable vector of parameters to be estimated. The log-linear representation produced by taking the log of trade flows $\tilde{f}_{ij} = ln(f_{ij})$:

$$\begin{aligned} \tilde{f}_{ij} &= h_{ij}\delta + \alpha + e_{ij} \\ e_{ij} &= u_i + v_j + \varepsilon_{ij} \end{aligned} \tag{3}$$

with u_i, v_j reflecting exporter and importer specific effects when the data are organized first by exporter and then by importing countries. In matrix/vector notation we can write:

$$y \quad = \quad H\delta + \alpha\iota_{N^2} + \Delta_u u + \Delta_v v + \varepsilon \tag{4}$$

where $y = vec(\tilde{F})$ is an $N^2 \times 1$ vector of the trade flow matrix logged and the matrices Δ_u, Δ_v are $N^2 \times N$, while the vectors u, v are $N \times 1$. The matrices Δ_u, Δ_v map elements from the $N \times 1$ vectors of country-specific exporter and importer effects in u, v to the appropriate origin-destination combination of countries reflected in the (i, j)th flow dyads in $vec(\tilde{F})$. The matrix $H = \begin{pmatrix} X_d & X_o & vec(C) \end{pmatrix}$, where LeSage and Pace (2008) show that $X_d = \iota_N \otimes X$, $X_o = X \otimes \iota_N$, with X being an $N \times 1$ vector of gross domestic product (gdp) for the countries, and ι_N an $N \times 1$ vector of ones. The Kronecker product ($\otimes$) applied to the country-level (gdp) vector strategically arranges country-level incomes to match the export-import dyads of the dependent variable vector that arises from vectorizing the flow matrix. The term $vec(C)$ is often simply a pairwise distance matrix vectorized as a proxy for trade costs between origin-destination dyads. A conformable vector δ contains parameters β_d, β_o and c associated with the variable vectors X_d, X_o and $vec(C)$.

As noted in the introduction, we can generalize proxies for trade costs to include not only distance ($vec(C)$), but also, for example, common borders and language. These binary indicator variables can be represented using $N \times N$ matrices W_b and W_l. The matrix H can be extended to include these indicator variables: $H = \begin{pmatrix} X_d & X_o & vec(C) & vec(W_b) & vec(W_l) \end{pmatrix}$, along with the extended vector δ.

A cross-sectional dependence specification that has been labeled the spatial Durbin model (SDM) is shown in Eq. (5), where we redefine $H = (X_d, X_o)$, $\beta = (\beta_d, \beta_o)'$ and $\theta = (\theta_d, \theta_o)'$:

$$\begin{aligned} y &= \rho W_c y + H\beta + W_c H\theta + \epsilon \\ \epsilon &\sim \mathcal{N}(0, \sigma^2 I_{N^2}). \end{aligned} \tag{5}$$

Here W_c is an $N \times N$ matrix reflecting a convex combination of the two weight matrices W_b and W_l.

The SDM specification allows for contextual effects as well as global spillovers from changes in country-level incomes reflected by elements contained in vectors X_d, X_o in the matrix H. This can be seen by noting that a change in income of country i, X_i, will have a partial derivative impact that involves the matrix inverse: $(I_{N^2} - \rho W_c)^{-1} = I_{N^2} + \rho W_c + \rho^2 W_c^2 + \ldots$ as shown in Eq. (6):

$$\partial y / \partial X_i = (I_{N^2} - \rho W_c)^{-1}(\beta_d + W_c\theta_d + \beta_o + W_c\theta_o). \tag{6}$$

LeSage and Thomas-Agnan (2015), and LeSage and Fischer (2016) provide specifics regarding the nature of these partial derivatives, but for our purposes we simply note that changes taking place in one country will have global spillover impacts on trade flows in neighboring countries, neighbors to the neighbors, and so on. There will also be feedback effects arising from matrices such as W_c^2, since the diagonal elements of this matrix contain non-zero elements. These reflect the fact that country i is a neighbor to its neighboring country j, or a second-order neighbor to itself.

3 Panel data models

In a panel setting, explanatory variables from the matrix H in the cross-sectional model in Eq. (5) that do not vary over time between countries must be eliminated. Variables such as distance $vec(C)$, and indicator variables for common borders, language, currency and other socio-cultural measures of similarity ($vec(W_b)$ and $vec(W_l)$) do not vary over time. Transformations such as the *within transformation* or that suggested by Lee and Yu (2010) can be used to eliminate country-specific effects. Given the motivation for cross-sectional dependence set forth above, a question arises whether time invariant factors reflect heterogeneity that is eliminated by fixed effects transformations.

We consider panel model specifications that use the ith column of the flow matrix $\tilde{F}$ representing exports from country i to all other countries as the dependent variable vector y over the $T = 38$ years from 1963 to 2000. We label the single explanatory variable (vector in the case of our application) X, containing (logged) gross domestic product per capita over the 38 years. Given our sample of $N = 74$ countries, this results in 74 different panel data models having dimension $(N-1) \times T$. [2]

An advantage of this approach is that we allow for different coefficient estimates for the model parameters for each of the N origin (exporting) countries and for a set of time invariant fixed effects for each destination (importing) country with respect to each origin country. This set of heterogenous coefficients contrasts with typical empirical trade panel data models that impose a restriction that coefficients on all explanatory variables are the same for all countries and time periods, with heterogeneity accounted for by the fixed effects parameters. Specifically, the conventional empirical trade panel models would stack the $N^2 \times T$ flow matrices as noted in the previous section and rely on a matrix H containing destination and origin incomes in the $N^2 \times 1$ vectors X_d, X_o. In our set of $(N-1) \times T$ panel models, we used the within transformation to eliminate country-specific effects.

[2] We exclude exports from country i to itself, which would be on the main diagonal of the trade flow matrix, since we have no information on intra-country flows, resulting in $N-1$.

Convex combinations of proximity structures

We focus on convex combinations of weight matrices that result in a single weight matrix reflecting multiple types of connectivity, where coefficients from the convex combination can be used for inference regarding the relative importance of each type of connectivity. For example, in our case of $L = 5$ weight matrices, $W_\ell, \ell = 1, \ldots, L$ reflecting L different types of dependence between our cross-section of countries:

$$W_c = \sum_{\ell=1}^{L-1} \gamma_\ell W_\ell + (1 - \sum_{\ell=1}^{L-1} \gamma_\ell) W_L, 0 \leq \gamma_\ell \leq 1, \sum_{\ell=1}^{L} \gamma_\ell = 1. \tag{7}$$

The matrix W_c reflects a convex combination of the L weight matrices, with the scalar parameters γ_ℓ indicating the relative importance assigned to each type of dependence. We wish to consider both conventional spatial dependence, which represents one type of cross-sectional dependence as well as multiple types of socio-cultural dependence (specifically, common currency, common language, trade agreements and colonial ties).

The spatial weight matrix $W_{\ell=1}$ reflects spatial proximity of countries (specifically some number of nearest neighbors). We rely on six nearest neighbors to form $W_{\ell=1}$. The other matrices $W_{\ell=2,\ldots,5}$, are constructed to reflect socio-cultural proximity based on: common currency $W_{\ell=2}$, common language $W_{\ell=3}$, membership in a trade agreement (excluding WTO membership) $W_{\ell=4}$, and direct historical colonial ties $W_{\ell=5}$.

There are some points to note regarding this approach. First, the matrices W_ℓ must be distinct, but can be highly correlated. If, for example, $W_{\ell=1} = W_{\ell=2}$, the parameters γ_1 and γ_2 will not be properly identified. Second, the matrices W_ℓ are row-normalized to have row-sums of unity, and zero diagonal elements. Zero diagonal elements exclude a country i from being a neighbor to itself. Row normalization ensures that the scalar cross-sectional dependence parameter ρ must be less than one, a conditional required for convergence of the infinite series expansion: $(I_N - \rho W_c)^{-1} = I_N + \rho W_c + \rho^2 W_c^2 + \ldots$.

Another point is that no individual row in the matrix W_c can contain only zeros. In the case of a spatial weight matrix $W_{\ell=1}$ based on some number (say) s nearest neighboring countries, all rows will by definition consist of non-zero elements. However, this results in non-zero rows in the matrix W_c only if $\gamma_{\ell=1}$ is non-zero. We allow zero values for the parameters γ_ℓ. To prevent zero rows in the matrix W_c, we restricted our sample of countries to those for which all $L = 5$ matrices had rows with non-zero elements. This resulted in elimination of countries such as South Korea and Japan that do not have a common language, common currency, direct colonial ties, etc. with any other country in the sample.

4 Computationally efficient expressions for the model

We extend the approach taken by Debarsy and LeSage (2017) that deals with cross-sectional specifications to the case of a static panel data setting. The static panel variant of the spatial Durbin model (SDM) that we wish to estimate is shown in Eq. (8), where each W_ℓ represents an $(N-1) \times (N-1)$ weight matrix whose main diagonal contains zero elements and row-sums of the off-diagonal elements equal to one, with N denoting the number of countries.[3] Non-zero (off-diagonal) weight matrix elements (i, j) of each W_ℓ reflect that observation j exhibits interaction with observation i, with different weight matrices describing different possible types of interaction (e.g., spatial, and different types of socio-cultural).

$$\begin{aligned}
y &= \rho(I_T \otimes W_c)y + X\beta + \sum_{\ell=1}^{L}(I_T \otimes W_\ell)X\theta_\ell + \varepsilon \\
&= \rho \sum_{\ell=1}^{L} \Big(I_T \otimes \gamma_\ell W_\ell \Big) y + X\beta + \sum_{\ell=1}^{L}(I_T \otimes W_\ell)X\theta_\ell + \varepsilon \qquad (8) \\
W_c &= \sum_{\ell=1}^{L} \gamma_\ell W_\ell, \quad 0 \leq \gamma_\ell < 1, \quad \sum_{\ell=1}^{L} \gamma_\ell = 1
\end{aligned}$$

with ρ denoting the scalar dependence parameter. The $(N-1)T \times 1$ vector y contains observations on exports (imports) from (to) country i to (from) all $(N-1)$ other countries for all time periods. These are organized with those for all (other) countries for the first time period, then all countries for the second time period, and so on. The $(N-1)T \times (N-1)T$ matrix $(I_T \otimes W_c)$ uses the Kronecker product to replicate the weight matrix for each time period. The $(N-1)T \times K$ matrix X in Eq. (8) contains the explanatory variables arranged in the same fashion as the dependent variable vector y, with β being the associated $K \times 1$ vector

[3]Note also that we have eliminated country-specific effects by applying the de-meaning transformation to the vector y and matrix X, but for notational simplicity we use y, X.

of parameters. In our case, the explanatory variable vector X reflects gdp pc (gross domestic product per capita) of the destination or origin countries in the case of exports or imports respectively, with θ being the associated $K \times 1$ vector of parameters. The $(N-1)T \times K$ matrices $(I_T \otimes W_1)X, (I_T \otimes W_2)X, \ldots, (I_T \otimes W_L)X$ reflect (logged) gdp pc in spatial neighbors to the origin/destination in the case of W_1, and countries with common currency, common language, trade agreements and direct colonial ties in the cases of W_2 to W_5. In the social networking literature these variable vectors are referred to as *contextual effects*, representing characteristics of peer groups defined by the matrix products $W_\ell X$ $(\ell = 1, ..., L)$ which create averages of peers' characteristics that might influence the outcomes vector y. In our model these variables allow for (average) income in spatial and socio-cultural neighbors produced by the matrix-vector products $W_\ell X$ to influence trade flows. Finally, the $(N-1)T \times 1$ vector ε represents a constant variance normally distributed disturbance term $(\varepsilon \sim \mathcal{N}(0, \sigma^2 I_{(N-1)T})$.

The model in Eq. (8) can be expressed as shown in Eq. (9), that is computationally convenient because it isolates the parameters $\rho, \gamma_\ell, \ell = 1, \ldots, L$ in the $(L+1) \times 1$ vector ω. We use: $\tilde{W}_\ell = (I_T \otimes W_\ell)$ in Eq. (9) to simplify notation.

$$
\begin{aligned}
\tilde{y}\omega &= Z\delta + \varepsilon && (9) \\
\tilde{y} &= \begin{pmatrix} y & \tilde{W}_1 y & \tilde{W}_2 y & \ldots & \tilde{W}_L y \end{pmatrix}, && (10) \\
\omega &= \begin{pmatrix} 1 & -\rho\gamma_1 & -\rho\gamma_2 & \ldots & -\rho\gamma_L \end{pmatrix}' = \begin{pmatrix} 1 & -\rho\Gamma \end{pmatrix}', \\
\Gamma &= \begin{pmatrix} \gamma_1 & \gamma_2 & \ldots & \gamma_L \end{pmatrix}', \\
\delta &= \begin{pmatrix} \beta & \theta_1 & \theta_2 & \ldots & \theta_L \end{pmatrix}', \; Z = \begin{pmatrix} X & \tilde{W}_1 X & \tilde{W}_2 X & \ldots & \tilde{W}_L X \end{pmatrix}. && (11)
\end{aligned}
$$

A related model labeled the spatial autoregressive (SAR) model can be constructed by redefining the matrix $Z = X$. This type of model excludes *contextual effects* embedded in the various types of neighboring countries income represented by the variable vectors $W_\ell X$.

The value of isolating the parameter vector ω is that this allows us to pre-calculate the $(N-1)T \times L$ matrix $\tilde{y}$ prior to beginning the Markov Chain Monte Carlo (MCMC) sampling loop. It also leads to quadratic form expressions for crucial terms that arise during MCMC sampling from the sequence of conditional distributions for the parameters. Quadratic forms produce computationally fast and efficient calculations.

5 The Markov Chain Monte Carlo estimation scheme

Here again, we extend the approach taken by Debarsy and LeSage (2017) for estimation in a cross-sectional setting to the case of the static panel data specification. Prior distributions along with conditional posterior distributions for the model parameters required to implement MCMC estimation of the SDM panel data specification in Eq. (8) are set forth here.

We rely on a normal prior for the parameters $\delta = \begin{pmatrix} \beta & \theta_1 & \dots & \theta_L \end{pmatrix}'$:

$$p(\delta) \sim \mathcal{N}(\bar{\delta}, \bar{\Sigma}_\delta) \tag{12}$$

where $\bar{\delta}$ is a $(K+L) \times 1$ vector of prior means and $\bar{\Sigma}_\delta$ is a $(K+L) \times (K+L)$ prior variance-covariance matrix.[4]

We employ a uniform prior for ρ since this scalar dependence parameter is constrained to lie in the open interval: $(-1, 1)$.[5] The constraint $(-1 < \rho < 1)$ is imposed during MCMC estimation using rejection sampling.

Since the parameters $\gamma_\ell, \ell = 1, \dots, L$ are a focus of inference, we do not impose a prior distribution on these parameters, but impose the closed interval $[0, 1]$ for γ_ℓ, $\ell = 1, \dots, L$ during MCMC estimation, and also impose $\sum_{\ell=1}^{L} \gamma_\ell = 1$, by setting $\gamma_L = (1 - \sum_{\ell=1}^{L-1} \gamma_\ell)$. We discuss how proposal values for the vector of parameters Γ are generated later.

For the parameter σ^2, we use an Inverse Gamma$(\bar{a}, \bar{b})$ distribution shown in Eq. (13). We note that as values of $\bar{a}, \bar{b} \to 0$, this prior distribution becomes uninformative, which might be important in applied practice since there would be little basis for assigning prior values for the parameter σ^2.

$$\begin{aligned} p(\sigma^2) &= \frac{\bar{b}^{\bar{a}}}{Gamma(\bar{a})} (\sigma^2)^{-(\bar{a}+1)} \exp(-\bar{b}/\sigma^2) \qquad (13) \\ \sigma^2 &> 0,\ \bar{a}, \bar{b} > 0. \end{aligned}$$

[4]We do not introduce an intercept vector and associated parameter since use of the within transformation to eliminate fixed effects precludes an intercept.

[5]A value of -1 is often used in practice as this ensures that the matrix inverse $(I_{(N-1)T} - \rho(I_T \otimes W_c))^{-1}$ exists. This has the advantage that we do not have to calculate the minimum eigenvalue of W_c which changes as a function of the values taken by γ.

As is traditional in the literature, we assume that priors for the parameters $\delta, \rho, \Gamma, \sigma^2$ are independent. Given these priors, we require the conditional distributions for the parameters $\delta, \sigma^2, \rho, \Gamma$ from which we sample to implement MCMC estimation. The conditional distribution for the parameters δ is multivariate normal with mean and variance-covariance shown in Eq. (14):

$$\begin{aligned} \delta|\rho, \sigma^2, \Gamma &= \mathcal{N}(\hat{\delta}, \hat{\Sigma}_\delta) \\ \hat{\delta} &= G^{-1}g \\ \Sigma_\delta &= \sigma^2 G^{-1} \\ G &= (Z'Z + \sigma^2\,\Sigma_\delta) \\ g &= (Z'\tilde{y}\,\omega + \Sigma_\delta\,\hat{\delta}). \end{aligned} \tag{14}$$

The conditional posterior for σ^2 (given δ, ρ, Γ) takes an Inverse Gamma (IG) form in Eq. (15), when we set the prior parameters $\bar{a} = \bar{b} = 0$:

$$\begin{aligned} p(\sigma^2|\beta, \omega) &\propto (\sigma^2)^{-(\frac{n}{2})}\exp\left(-\frac{1}{2\sigma^2}\omega'(e'e)\omega\right) \\ e &= (\tilde{y} - Z\delta) \\ \delta &= (Z'Z)^{-1}(Z'\tilde{y}) \\ &\sim IG(\tilde{a}, \tilde{b}) \\ \tilde{a} &= n/2 \\ \tilde{b} &= \frac{1}{2}\omega'(e'e)\omega \end{aligned} \tag{15}$$

The (log) conditional posterior for ρ (given δ, Γ, σ^2) has the form in Eq. (16), where we use $T\ln|I_{N-1} - \rho W_c(\Gamma)|$ to show that the log-determinant term in this model depends on the vector Γ. For example, considering a convex combination of three matrices, we need to calculate: $T\ln|I_N - \rho W_c(\gamma)| = T\ln|I_N - \rho(\gamma_1 W_1 + \gamma_2 W_2 + \gamma_3 W_3)|$ with $\gamma_3 = 1 - \gamma_1 - \gamma_2$. (We provide details regarding a computationally efficient approach to calculating the log-determinant term in the next section.)

$$ln\ p(\rho|\delta, \Gamma, \sigma^2) \propto -\frac{(N-1)T}{2} ln\,\sigma^2 + Tln|I_{N-1} - \rho W_c(\Gamma)| - \frac{1}{2\sigma^2}\omega(\rho)'(e'e)\omega(\rho) \tag{16}$$

where we use the expression $\omega(\rho)$ to indicate that only the parameter ρ in the vector ω varies, with the parameter vector Γ fixed.

This distribution does not reflect a known form as in the case of the conditional distributions for δ and σ^2. We sample the parameter ρ from this conditional distribution using a Metropolis-Hastings sampling approach. Details are described in the next section where we outline our approach to avoid repeated calculation of the log-determinant term in this conditional distribution.

The (log) conditional posterior for Γ (given δ, ρ, σ^2) takes the form in Eq. (17), where we also have a log-determinant that depends on values taken by the vector Γ. We use the expressions $\omega(\Gamma)$ to indicate that these parameter vectors depend on Γ with the parameter vectors ρ fixed.

$$\begin{aligned} ln\ p(\Gamma|\delta, \rho, \sigma^2) &\propto -\frac{(N-1)T}{2} ln\, \sigma^2 + Tln|I_{N-1} - \rho W_c(\Gamma)| \\ &- \frac{1}{2\sigma^2}\omega(\Gamma)'(e'e)\omega(\Gamma). \end{aligned} \tag{17}$$

As in the case of the conditional distribution for ρ, this distribution does not reflect a known form. We sample the parameter vector Γ as a block from this conditional distribution using a reversible jump procedure to produce proposal values for the vector Γ in conjunction with Metropolis-Hastings sampling. Details are described in the next section.

6 A computationally efficient approach based on trace approximations

Debarsy and LeSage (2017) point to three computational challenges arising for this type of model where the weight matrix W_c is a function of estimated parameters γ_ℓ. One is that the log-determinant term in the conditional distributions for ρ and Γ in Eqs. (16) and (17) cannot be pre-calculated over a range of values for the dependence parameter ρ as is conventionally done in single weight matrix spatial regression models. A second issue relates to dealing with the restriction imposed on the parameters $\sum_{\ell=1}^{L} \gamma_\ell = 1$. The third challenge arises when calculating measures of dispersion for the partial derivatives $\partial y / \partial X$ that LeSage and Pace (2009) label *effects estimates*. An empirical measure of dispersion for the effects is typically constructed by evaluating the partial derivatives using a large number (say 1,000) MCMC draws for the parameters.[6] The expressions for the partial derivatives involve the inverse of an $(N-1)\times(N-1)$ matrix. For the case of a single weight matrix, LeSage and Pace (2009) show how to use a trace approximation to avoid calculating the matrix inverse thousands of times, but this approach does not apply to the model developed here.

In section 6.1, a Taylor series approximation for the log-determinant term is set forth. The log-determinant term arises in the conditional distributions [see Eqs. (16) and (17)] required to sample the dependence parameter ρ and the parameters γ_ℓ, $\ell = 1, \ldots, L$ that serve as weights in the convex combination. Section 6.2 describes a reversible jump approach to block sampling the parameters γ_ℓ, $\ell = 1, \ldots, L$. Calculation of the effects estimates which represent partial derivatives of the dependent variable with respect to changes in the explanatory variables is the subject of section 6.3.

6.1 A Taylor series approximation for the log-determinant

Pace and LeSage (2002) set forth a Taylor series approximation for the log-determinant of a matrix like our expression: $\ln|I_{N-1} - \rho\tilde{W}_c|$. They show that for a *symmetric* non-negative weight matrix $\tilde{W}_c$ with eigenvalues $\lambda_{\min} \geq -1, \lambda_{\max} \leq 1$, and $1/\lambda_{\min} < \rho < 1$, and $tr(\tilde{W}_c) = 0$, where tr represents the trace:

$$\ln|I_{N-1} - \rho\tilde{W}_c| = -\sum_{j=1}^{\infty} \frac{\rho^j W_c^j}{j} \tag{18}$$

$$\simeq -\sum_{j=1}^{q} \frac{\rho^j tr(W_c^j)}{j}. \tag{19}$$

Golub and van Loan (1996, p. 566) provide the expression in Eq. (18), while Pace and LeSage (2002) note that due to the linearity of the trace operator we have expression (19). We note that the 1st-order trace involves $tr(W_c)$ which is zero for any convex combination of weight matrices that have zero diagonal elements. For symmetric matrices W_ℓ, we can express the 2nd-order trace as a quadratic form in Eq. (20) involving the vector of parameters Γ and all

[6] In the case of maximum likelihood estimation, parameters (say 1,000) are drawn from a normal distribution using the mean estimates and estimated covariance matrix based on a numerical or analytical Hessian.

pairwise multiplications of the individual matrices W_ℓ as shown in Eq. (21):

$$
\begin{aligned}
tr(W_c^2) &= \Gamma' Q^2 \Gamma && (20)\\
\Gamma' &= \begin{pmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_L \end{pmatrix} \\
Q^2 &= \begin{pmatrix} W_1 \times W_1 & W_1 \times W_2 & \dots & W_1 \times W_L \\ W_2 \times W_1 & W_2 \times W_2 & \dots & W_2 \times W_L \\ \vdots & & & \\ W_L \times W_1 & W_L \times W_2 & \dots & W_L \times W_L \end{pmatrix}. && (21)
\end{aligned}
$$

This formulation separates the parameters in the vector Γ from the matrix of traces, which allows pre-calculation of the matrix of traces for a given set of weight matrices W_ℓ prior to MCMC sampling. For the case of asymmetric matrices we use matrix products $\sum_i^L \sum_j^L W_i \odot W_j'$. We note that row-normalized weight matrices would be an example of asymmetric matrices. Our socio-cultural weight matrices are by definition symmetric, because countries i and j with common language, common currency, and so on, would result in countries j and i having common language, common currency, and so on.

Debarsy and LeSage (2017) emphasize that a more efficient computational expression is $(\Gamma \otimes \Gamma) vec(Q^2)$, where $\otimes$ is the Kronecker product and vec the operator that stacks the columns of the matrix Q^2. Using this approach leads to a similar expression for the 3rd-order trace, which in the case of $L = 2$ takes the form involving L^3 matrix products:

$$
tr(W_c^3) = (\Gamma \otimes \Gamma) \otimes \Gamma \ vec(Q^3) \qquad (22)
$$

where again, we can use sums of matrix products to produce the L^3 matrix products required:

$$
Q^3_{ijk} = \sum_i^L \sum_j^L \sum_k^L (W_i \times W_j) \odot W_k. \qquad (23)
$$

We rely on a 4th-order Taylor series approximation, since Debarsy and LeSage (2017) provide results from a Monte Carlo experiment showing that this produces the desired accuracy in a cross-sectional model setting.

A fourth-order Taylor series approximation to the log-determinant $T\ln|I_{N-1} - \rho W_c|$ takes the form in Eq. (24).

$$\begin{aligned} T\,ln|I_{N-1} - \rho W_c| \;\; &\simeq \;\; T(-\rho^2(\Gamma \otimes \Gamma)vec(\frac{Q^2}{2}) \\ &\quad -\rho^3(\Gamma \otimes \Gamma) \otimes \Gamma \; vec(\frac{Q^3}{3}) \\ &\quad -\rho^4((\Gamma \otimes \Gamma) \otimes \Gamma) \otimes \Gamma \; vec(\frac{Q^4}{4})). \end{aligned} \tag{24}$$

The conditional distribution for the parameter ρ consists of the log-determinant term as well as a term involving the sum of squared errors: $(1/2\sigma^2)\omega'(e'e)\omega$. Since the conditional distribution is evaluated twice when carrying out the Metropolis-Hastings step for sampling the parameter ρ, once at the current value of ρ (which we label ρ^c) and a second time at the proposed value (which we label ρ^p), the quadratic forms plus the Taylor series trace approximation to the log-determinant allow for rapid calculations.[7]

The (log) conditional distribution for ρ is shown in Eq. (25), where the expression $\omega(\rho)$ indicates that only the parameter ρ in the vector ω varies, with elements in the vector Γ fixed:

$$\begin{aligned} ln\; p(\rho|\delta, \Gamma, \sigma^2) \;\; &\propto \;\; -\frac{N-1}{2} ln\, \sigma^2 T[-\rho^2(\Gamma \otimes \Gamma)vec(\frac{Q^2}{2}) - \rho^3(\Gamma \otimes \Gamma) \otimes \Gamma \; vec(\frac{Q^3}{3}) \\ &\quad -\rho^4((\Gamma \otimes \Gamma) \otimes \Gamma) \otimes \Gamma \; vec(\frac{Q^4}{4})] - \frac{1}{2\sigma^2}\omega(\rho)'(e'e)\omega(\rho). \end{aligned} \tag{25}$$

The current value of ρ^c is evaluated in Eq. (25) as well as a proposal value ρ^p. The proposal value is generated using a tuned random-walk procedure: $\rho^p = \rho^c + \kappa\mathcal{N}(0, 1)$, where κ is a tuning parameter and $\mathcal{N}(0, 1)$ denotes a standard normal distribution. The tuning parameter is adjusted based on monitoring the acceptance rates with κ adjusted downward using $\kappa' = \kappa/1.1$ if the acceptance rate falls below 40%, and adjusted upward using $\kappa' = (1.1)\kappa$ when the acceptance rate rises about 60% (see LeSage and Pace, 2009, p. 137). The (non-logged) conditional distributions are then used in expression (26) to calculate a Metropolis-Hastings acceptance probability ψ_{MH},

[7] Note also that we pre-compute $\tilde{y}$ prior to MCMC sampling.

where we use $(\cdot)$ to denote the conditioning parameters $(\delta, \Gamma, \sigma^2)$:

$$\begin{aligned} \psi_{MH}(\rho^c, \rho^p) &= \min[1, \frac{p(\rho^p|\cdot)}{p(\rho^c|\cdot)}] \\ &= \min\left[1, \exp((ln\ p(\rho^p|\cdot) - ln\ p(\rho^c|\cdot))\right]. \end{aligned} \tag{26}$$

If $(p(\rho^p|\cdot) - p(\rho^c|\cdot)) > \exp(1))$, the Metropolis-Hastings probability (MH_p) is set to one, otherwise, MH_p is calculated using: $\psi_{MH}(\rho^c, \rho^p)$. This probability (MH_p) is compared to a uniform$(0,1)$ random draw to make the accept/reject decision based on (uniform$(0,1) < MH_p) \rightarrow$ accept), otherwise reject.

6.2 A reversible jump approach to block sampling Γ

A second computational challenge for MCMC estimation of the model is sampling parameters in the vector Γ, which must sum to one. We rely on a block-sampling approach set forth in Debarsy and LeSage (2017). This involves a proposal vector of candidate values for $\gamma_\ell, \ell = 1, 2, \ldots, L-1$, with $\gamma_L = 1 - \sum_{\ell=1}^{L-1}$. Since a vector of proposal values are produced, it is easy to impose the restriction that $\sum_\ell \gamma_\ell = 1$. The conditional distributions for the current and proposed vectors that we label Γ^c, Γ^p are evaluated with a Metropolis-Hastings step used to either accept or reject the newly proposed vector Γ^p. Block sampling the parameter vector Γ has the virtue that accepted vectors will obey the summing up restriction and reduce autocorrelation in the MCMC draws for these parameters. However, block sampling is known to produce lower acceptance rates which may require more MCMC draws in order to collect a sufficiently large sample of draws for posterior inference regarding Γ.

Debarsy and LeSage (2017) use a reversible jump procedure to produce the proposal values for the vector Γ. This involves (for each $\gamma_\ell, \ell = 1, \ldots, L-1$) a three-headed coin flip. By this we mean a uniform random number on the open interval *coin flip* $= U(0,1)$, with head #1 equal to a value $\leq 1/3$, head #2 a value $> 1/3$ and $\leq 2/3$, and head #3 equal to a value $> 2/3$ and smaller than one. Given a head #1 result, we set a proposal for γ_ℓ^p using a uniform random draw on the open interval $(0 < \gamma_\ell^c)$, the current value. A head #2 results in setting the proposal value equal to the current value $(\gamma_\ell^p = \gamma_\ell^c)$, while a head #3 selects a proposal value based on a uniform random draw on the open interval ($\gamma_\ell^c < 1$). [8]

The (non-logged) conditional distributions in expression (27) are used to calculate a Metropolis-Hastings acceptance probability, where we use $(\cdot)$ to denote the conditioning parameters (δ, ρ, σ^2):

$$\psi_{MH}(\Gamma^c, \Gamma^p) \quad = \quad \min\left[1, \exp((ln\ p(\Gamma^p|\cdot) - ln\ p(\Gamma^c|\cdot))\right] \tag{27}$$

The (log) conditional posterior for the (say the proposal) vector Γ^p (given δ, ρ, σ^2) in Eq. (28) can be rapidly evaluated using the log-determinant approximation and the quadratic forms representation of the sum-of-squared errors. We use $\omega(\Gamma^p)$ in Eq. (28) to indicate that only the vector Γ changes in the vector ω, with the value of ρ fixed.

$$\begin{aligned} ln\ p(\Gamma^p|\delta, \rho, \sigma^2) \quad \propto \quad & -\frac{(N-1)T}{2}\ln\sigma^2 + T[-\rho^2(\Gamma^p \otimes \Gamma^p)vec(\frac{Q^2}{2}) - \rho^3(\Gamma^p \otimes \Gamma^p) \otimes \Gamma^p\ vec(\frac{Q^3}{3}) \\ & -\rho^4((\Gamma^p \otimes \Gamma^p) \otimes \Gamma^p) \otimes \Gamma^p\ vec(\frac{Q^4}{4})] - \frac{1}{2\sigma^2}\omega(\Gamma^p)'(e'e)\omega(\Gamma^p) \end{aligned} \tag{28}$$

There are further computational gains from calculating some matrices prior to MCMC sampling.

6.3 Calculating effects estimates

The third computational challenge tackled by Debarsy and LeSage (2017) relates to constructing an empirical posterior distribution for the effects estimates representing the model partial derivatives. LeSage and Pace (2009) point out that for the case of (our) SDM model, partial derivatives take the form in Eq. (29) for the single explanatory variable vector X (logged gdp pc). They propose scalar summary measures of the own- and cross-partial derivatives that they label *direct* and *indirect* effects, shown in Eqs. (30) and (32), where *tr* represents the trace

[8] See Debarsy and LeSage (2017) for a discussion of the reversible jump nature of this procedure.

operator and ι_{N-1} is an $(N-1) \times 1$ vector of ones.[9]

$$\begin{aligned}
\partial y/\partial X &= S(W_c) &\quad (29)\\
S(W_c) &= (I_{N-1} - \rho W_c)^{-1}(I_{N-1}\beta + W_c\theta)\\
&= (I_{N-1}\beta + W_c\theta + \rho W_c(I_{N-1}\beta + W_c\theta) + \rho^2 W_c^2(I_{N-1}\beta + W_c\theta)\ldots\\
\bar{M}_{direct} &= (N-1)^{-1} tr[S(W_c)] &\quad (30)\\
\bar{M}_{total} &= (N-1)^{-1}\iota_{N-1}' S(W_c)\iota_{N-1} &\quad (31)\\
\bar{M}_{indirect} &= \bar{M}_{total} - \bar{M}_{direct} &\quad (32)\\
W_c &= \sum_{\ell=1}^{L} \gamma_\ell W_\ell.
\end{aligned}$$

While the expressions in Eqs. (30), (31) and (32) produce point estimates for the scalar summary measures of effects (own- and cross-partial derivatives) used to interpret the impact of changes in the SDM model explanatory variables on dependent variable outcomes, we also require measures of dispersion for the purpose of statistical tests regarding the significance of these effects. Use of an empirical distribution constructed by simulating the non-linear expressions in Eq. (29) using (say 1,000) draws from the posterior distribution of the underlying parameters $\rho, \beta_r, \theta_r, \gamma_\ell, \ell = 1, \ldots, L$ is suggested by LeSage and Pace (2009),

Note that a naive approach to such a simulation-based empirical distribution would require calculation of the $(N-1) \times (N-1)$ matrix inverse $(I_{N-1} - \rho W_c)^{-1}$ a large number of times, for varying values of the parameters $\rho, \gamma_\ell, \ell = 1, \ldots, L$, which would be computationally intensive.

The required quantity for constructing the empirical distribution of the effects is $tr(S(W_c))$, which can be estimated without a great deal of computational effort (see LeSage and Pace 2009 for details). In the case described in LeSage and Pace (2009), the SDM model relies on a single weight matrix W, allowing use of estimated traces $tr(W^2)/(N-1), tr(W^3)/(N-1), \ldots, tr(W^q)/(N-1)$ calculated once prior to simulation of the effects estimates. This allows simulation of the empirical distribution for the effects estimates using only vector products involving draws of the parameters ρ, δ taken from their posterior distributions.

[9]Elhorst (2013) points out that the effects for static spatial panel data models such as ours are the same as those developed by LeSage and Pace (2009) for the cross-sectional model, because the weight matrices and parameters do not vary over time periods.

Our situation differs because the matrix W_c depends on estimated parameters $\gamma_\ell, \ell = 1, \ldots, L$ ruling out use of estimated traces calculated prior to the simulation. We could rely on posterior means for γ_ℓ, i.e. $\bar{\gamma}_\ell$, to create a single matrix $\hat{W}_c(\bar{\gamma}_\ell)$, for which estimated traces could be calculated prior to simulation. However, this would ignore stochastic variation in the effects estimates that arise from the fact that there is uncertainty regarding the parameters γ_ℓ. Ideally, we would like to use draws for the γ_ℓ parameters from their posterior distributions when simulating the empirical distribution of effects estimates.

Debarsy and LeSage (2017) point out that since we have already calculated trace expressions for $j = 2, 3, 4$ in Eq. (19) to produce the Taylor series approximation to the log-determinant term based on the quadratic forms in Eq. (33), these can be used to replace low-order traces estimated based on posterior means $(\bar{\gamma}_\ell)$ used to construct a single matrix W_c. Higher-order traces decline in magnitude, so low-order traces are most important for accurate estimates of the effects.

Specifically, their approach estimates $q = 100$ traces using the approach of LeSage and Pace (2009), based on a single weight matrix $\hat{W}_c = \sum_{\ell=1}^{L} \bar{\gamma}_\ell W_\ell$, constructed using posterior means for γ_ℓ, then replace the estimated 1st-order trace with zero (a known value), and the 2nd- through 4th-order traces with terms shown in Eq. (33). The MCMC sampled parameters Γ are used in the expressions (33) during the simulation that produces the empirical distribution of effects estimates. Debarsy and LeSage (2017) note that this incorporates uncertainty regarding the parameters γ_ℓ for low-order traces since they are using MCMC draws for these parameters. They argue that since higher-order terms involve increasingly smaller magnitudes of the parameters ρ and Γ, low-order traces are most important for accurate estimates of the effects.

$$\begin{aligned} tr(W_c^2) &= (\Gamma \otimes \Gamma) vec(Q^2) \\ tr(W_c^3) &= (\Gamma \otimes \Gamma) \otimes \Gamma \ vec(Q^3) \\ tr(W_c^4) &= ((\Gamma \otimes \Gamma) \otimes \Gamma) \otimes \Gamma \ vec(Q^4). \end{aligned} \tag{33}$$

Of course, this is a computational compromise between calculating an empirical distribution for the effects estimates based on the exact formula which would require thousands of evaluation of the $(N-1) \times (N-1)$ matrix inverse. A series of Monte Carlo experiments reported by Debarsy and LeSage (2017) show that this approach produces effects estimates with very little bias except in cases where the level of spatial dependence is very high (e.g., values of $\rho \leq -0.9$ or $\rho \geq 0.9$).

7 Application of the cross-section dependence panel models

We consider panel model specifications that use the ith column (ith row) of the flow matrix representing exports (imports) from (to) country i to (from) all other $(N-1)$ countries j as the dependent variable vector y over the 38 years from 1963 to 2000. The explanatory variable is (logged) gross domestic product per capita lagged one year to cover the period from 1962 to 1999. The trade flows are from Feenstra et al. (2005), while the gdp data at market prices (current US$) and population data come from World Bank's (2002) World Development Indicators. A usable sample of 74 countries (see Table A.1 in Appendix A) was constructed for which gdp, population and trade flows were available over the 38 years.[10]

Given our sample of 74 countries, this results in 74 different panel data models for imports and 74 models for exports having dimension $(N-1) \times T$, where $N = 74$ and $T = 38$.

This approach allows panel estimation based on the T time periods for each country's exports/imports relationship such that we have heterogeneous coefficients across countries. Specifically, different (country-specific) dependence parameters ρ_i reflect different levels of dependence, different responses δ_i to own- and neighboring countries income, and parameters $\gamma_\ell, \ell = 1, \ldots, L$ as well as different noise variance estimates $\sigma^2_{\varepsilon,i}$. The specification also implicitly allows for fixed effects between each dyad of countries, since there will be a set of $N-1$ fixed effects for each country i's exports to (imports from) all other countries j. As already noted, we eliminated country-specific effects using the within-transformation to eliminate these effects prior to estimation.

7.1 Evidence of cross-sectional dependence

The first question we examine is whether trade flows exhibit cross-sectional dependence, which is a different phenomenon than heterogeneity modeled by the fixed effects transformations. In the

[10]In addition, we eliminated countries from our sample that had one or more zero rows in any of the five weight matrices. As noted earlier, this is necessary to ensure that the matrix W_c does not contain zero rows, when we allow individual $\gamma_\ell, \ell = 1, \ldots, L$ parameters to take values of zero. This resulted in a few countries such as South Korea and Japan for which data was available to be excluded from our sample.

presence of cross-sectional dependence, estimates from conventional models that ignore cross-sectional dependence can be shown to be biased and inconsistent.

The presence of cross-sectional dependence also implies spillover impacts arising from changes in neighboring countries $j \neq i$ income on country i's trade flows. In our model, neighbors are defined broadly to include both spatial neighbors as well as socio-cultural neighbors. Specifically, changes in income of countries j that have spatial, common language, currency, trade agreements, or colonial ties with country i will impact export or import flows in the SAR model, provided that the scalar dependence parameter ρ is different from zero and the parameter β is non-zero. In the case of the SDM model, the scalar dependence parameter ρ could be zero but there will still be spillover impacts if the parameters $\theta_\ell, \ell = 1, \ldots, L$ are non-zero.

Figure 1 shows a histogram of the distribution of estimates for the 74 different scalar dependence parameters ρ from the SAR model estimates, and Figure 2 shows that for the SDM model estimates of ρ. Recall, we estimate 74 export and 74 import models, and the figures show histograms for both import and export model estimates. From the figures, it should be clear that all 148 sets of SAR and 148 sets of SDM estimates are positive. They were also all statistically different from zero based on lower 0.05 and upper 0.95 credible intervals constructed from the (empirical) posterior distribution based on MCMC draws.

(Fig.1 and Fig.2 to be positioned here)

Table 1 shows the mean value for ρ over all 74 countries along with standard deviations of the distribution across countries and a $t-$statistic constructed using the mean divided by the standard deviation. These results are consistent with the notion that we have a distribution of cross-sectional dependence estimates for our sample of 74 countries that is different from zero.

(Table 1 to be positioned here)

We note that estimates for the parameters γ_ℓ that are discussed in the next section are not well-identified for values of ρ near zero. Intuitively, in the face of no cross-sectional dependence estimates of the relative importance/weights assigned to different types of cross-sectional connectivity structures are meaningless. Since estimates of the cross-sectional dependence parameters ρ were positive and different from zero for all countries, we can appropriately turn attention to the estimates for γ_ℓ that provide an indication of the relative importance of each of the five types of dependence.

7.2 Relative importance of spatial and socio-cultural connections

As motivated, the relative sizes of the parameter estimates for γ_ℓ allow us to draw conclusions about what types of connectivity are important. Figure 3 shows a histogram of these five sets of parameter estimates for the 74 countries determined using the import flows SDM models. Figure 4 shows these estimates for the export flows SDM models.

(Fig.3 and Fig.4 to be positioned here)

For the import models we see a relatively large number of countries (48) where estimates for γ associated with common currency take on small values less than 0.1, and the same is true for common language where we see 30 countries in this range of small values. In the case of export model estimates shown in Figure 4, we also see evidence that γ estimates associated with common language and currency weight matrices take on small values less than 0.1 for a large number (over 50) of the 74 countries.

A more formal approach to examining this issue involves counting countries where lower 0.05 bounds of the (truncated) distribution of MCMC draws for the parameters γ is greater than zero.[11] Table 2 shows these counts of countries for both the SAR and SDM models of import and export flows. From the table we see that for the case of the SDM models, spatial dependence and colonial ties were among the most important types of dependence in both import and export models. Of the 74 countries the γ parameters on W-space were non-zero in 58 and 59 countries for import and export models, respectively. In the case of colonial ties, there were 52 countries with non-zero weight placed on this type of dependence for the import models and 61 countries for the export models. This suggests that colonial ties are slightly more important for explaining variation in export flows than import flows.

(Table 2 to be positioned here)

[11] Technically, although we allow for the open interval ($0 < \gamma < 1$), we consider a lower 0.05 value above 0.01 for the MCMC draws to be non-zero.

For the SDM models, common currency was the least important type of dependence for import models, since only 18 countries had non-zero γ estimates, and common language for export models with 14 non-zero countries. Common currency was next least important for export models, while import models treated common language more importantly with 38 non-zero countries. Of course, imported consumer goods may require common language marketing labels and instruction manuals, partially explaining this type of result. The existence of trade agreements between countries seems to be important for both imports and exports in slightly more than half of the 74 countries examined (non-zero estimates for 37 and 41 countries respectively).

A similar pattern arose for the counts arising from the SAR models as discussed for the SDM models.

Table 3 shows the means and standard deviations σ_γ for the 74 countries posterior estimates of γ_ℓ, for both the SAR and SDM models of import and export. We note that since the posterior means across $\gamma_\ell, \ell = 1, \ldots, 5$ sum to unity for each country, the means across our sample of 74 countries reported in the table also sum to one.

(Table 3 to be positioned here)

The magnitudes reported reflect the patterns of counts from Table 3, with average γ values for the spatial weights being the largest (around 0.33), for SAR and SDM models of both imports and exports. In the case of import models, the second most important type of connectivity between countries was the existence of trade agreements (except the SDM export model), with an average value around 0.23 for both SAR and SDM models of both imports and exports. In the case of SAR import models estimates give roughly equal weight of 0.13 to the remaining three types of connectivity structures (common currency, language and colonial ties) with SDM import models also roughly equal with slightly less weight given to common currency. We also see agreement between the SAR and SDM models with regard to the importance of the remaining three types of connectivity (common currency, language and colonial ties) for exports. Trade agreements and colonial ties were most important (around 0.23) and common language least important (around 0.07).

A more complete picture of the γ_ℓ weights assigned to the various types of dependence is provided in Table 4 to Table 8. Country-level estimates for each of the five γ_ℓ parameters are sorted from low-to-high. It is important to note when considering the magnitudes of these estimates that simultaneous cross-sectional dependence implies that changes in income in country i will impact neighboring countries (first-order nodes in the connectivity structure/network) as well as higher-order neighboring nodes. That is, neighbors to the neighboring countries, neighbors to the neighbors of the neighbors, and so on, with the magnitude of impact declining for higher-order neighboring relations.

(Table 4 - Table 8 to be positioned here)

An implication of this is that (for example) colonial ties could reflect an important connectivity structure for countries like Sweden or Finland who do not have immediate (first-order) colonial ties. Nonetheless, cross-sectional dependence suggests that if colonial ties are important for major trading partners of Sweden or Finland, then this type of connectivity structure would also be important (receive a large γ estimate) for Sweden or Finland. Similar statements could be made about other types of connectivity structures, important higher-order links/nodes in the network of trading partners can mean that these connectivity structures represent an important source of cross-sectional dependence.

An unfortunate aspect of models such as that set forth here that rely on multiple types of connectivity (simultaneous dependence weight matrices) is that we cannot separate out the spillover/network impacts arising from each type of connection. This can be seen by considering the matrix inverse: $(I_N - \rho W_c)^{-1} = I_N + \rho W_c + \rho^2 W_c^2 + \dots$ which will contain numerous cross-products involving the different matrices $W_\ell, \ell = 1, \dots, L$. Higher-order powers will in general involve increasing larger matrix cross-products. The spirit of the model specification is that (say) spatial proximity to countries whose trade patterns rely heavily on (say) colonial ties might lead to multiple transmission channels that ultimately impact the observed patterns of trade flows.

7.3 Empirical estimates of bias from ignoring cross-sectional dependence

Ignoring spatial and socio-cultural dependence when estimating empirical trade flow models will lead to bias in estimates of the impact arising from income on trade flows. The magnitude of the bias can be quantified by examining the size and significance of the indirect effects estimates from the SAR and SDM model specifications. The size of the indirect effects depends on the magnitude of the dependence parameter ρ as well as the coefficient on income β in the case of the SAR specification. Intuitively, in cases where there is an absence of cross-sectional dependence ($\rho = 0$) we will not see a large amount of bias.

For the SDM specification, the size of indirect effects is determined by the dependence parameter ρ, the coefficient on income β as well as coefficients $\theta_\ell, \ell = 1, \ldots, L$. Here even in the absence of cross-sectional dependence, non-zero values for the parameters θ_ℓ would indicate omitted variable bias arising from contextual effects ignored by traditional models that do not include explanatory variables measuring these influences. Cross-sectional dependence reflects the fact that trade takes place in the context of a world-wide network of flows.

Since conventional trade models ignore cross-sectional dependence of the type captured by the SAR specification by assuming that $\rho = 0$, this implies an assumption of no spillovers (indirect effects of zero). If the SAR specification is consistent with the data, omitted variables bias will arise, and estimates of the coefficients representing the impact of country-level income on trade flows will likely overstate this impact by inappropriately attributing variation in trade flows to own-country income. In cases where the SDM specification is most consistent with the data, conventional models ignore the influence of neighboring countries income, where neighboring countries are broadly defined to include spatial as well as socio-cultural neighbors. In cases where the SDM specification is the data generating process, bias in conventional models can be attributed to ignoring both interaction between countries (assuming $\rho = 0$) as well as contextual effects (assuming $\theta_\ell = 0$).

(Fig.5 and Fig.6 to be positioned here)

Figure 5 shows a frequency distribution of the posterior mean indirect effects estimates from the SAR models of imports and exports across the 74 countries, and Figure 6 displays these effects for the SDM models. For the SAR import model we see 17 countries where indirect effects are near zero and in the case of the export model 14 countries with near zero indirect effects. Remaining countries exhibit positive spillovers reflecting the magnitude of bias that would arise from ignoring cross-sectional dependence. In the case of the SDM specification there are 17 of the 74 countries with (near) zero spillovers in the case of both the import and export models, with mostly positive spillovers.

8 Closing remarks

We raise questions about the role played by time invariant country-specific factors in explaining variation in trade flows. These are typically viewed and modeled using fixed effects or transformations to capture the heterogeneity impact of these in panel data models of trade flows.

Our findings indicate that conventional approaches to eliminating fixed effects associated with time invariant factors leave a great deal of variation in trade flows unexplained. This unexplained variation takes the form of: (i) cross-sectional dependence of trade flows on neighboring country flows, and/or (ii) contextual effects from neighboring country income levels. Using data transformed to eliminate time invariant fixed effects, we use a panel data extension of the Bayesian SAR and SDM models set forth in Debarsy and LeSage (2017) to examine the question of cross-sectional dependence and contextual effects using a panel of imports and exports from a sample of 74 countries over the 38 year period from 1963 to 2000. Specifically, we consider 148 different panel data models, 74 models for imports of each country from all other 73 countries over the 38 year period in our sample, and another set of 74 panel data models for exports from each country to all other 73 countries, covering the 38 year time period.

The SAR and SDM models utilize a convex combination of different types of connectivity between countries. We consider: spatial proximity, common currency and language connections, trade agreements and colonial ties. The models produce estimated weights for each of the five types of connectivity that sum to unity, allowing a posterior inference regarding the relative importance of the various types of connectivity. Our findings indicate that the most important type of connectivity is spatial proximity to neighboring countries, with the next most important types of connectivity being trade agreements and colonial ties. Common currency and language represent the least important connections between countries.

The spatial autoregressive (SAR) and spatial Durbin model (SDM) specifications capture simultaneous cross-sectional dependence between trade flows, with significant cross-sectional dependence pointing to biased and inconsistent estimates for model specifications that ignore the presence of this type of dependence. Simultaneous cross-sectional dependence implies spillovers from changes in one country's income to other countries, with the pattern of impacts falling on *neighboring countries*. In our model, that utilizes a convex combination of different types of connectivity, neighboring countries are broadly defined to include countries: (i) located nearby in space, having (ii) common currency, (iii) common language, (iv) trade agreements, or (v) colonial ties. The spillovers can impact immediately neighboring countries, neighbors to the neighboring countries, neighbors to the neighbors of the neighbors, and so on, with impact declining for higher-order neighboring relations.

The implications of our findings are twofold. One is that conventional treatment of generalized distance factors such as common language, free trade and stronger forms of agreements, common currency, and so on, as time invariant sources of heterogeneity in empirical panel trade model specifications ignores potential cross-sectional dependence and/or contextual effects (characteristics of neighboring countries). We explored the magnitude of bias that arises from this problem. A second implication is that from a theoretical perspective socio-cultural proximity of countries seems as important as pure geographical proximity. Our estimates point to spatial proximity receiving around 1/3 weight and socio-cultural proximity around 2/3 weight.

The results presented here suggest more attention be given to panel model specifications that allow for cross-sectional dependence in trade flows, as well as models that incorporate neighboring country characteristics. This suggests more emphasis on theoretical and empirical models of the type introduced by Lebreton and Roi (2011), Koch and LeSage (2015) for bilateral trade flows, LeSage and Pace (2008), Baltagi, Egger and Pfaffermayr (2007, 2008) for bilateral migration, and Behrens, Ertur and Koch (2012) for foreign direct investment.

APPENDIX A

Table A.1: List of countries

Algeria	Costa Rica	Jamaica	Saint Kitts and Nevis
Australia	Denmark	Kenya	Senegal
Austria	Dominican Rep.	Madagascar	Sierra Leone
Bahamas	Ecuador	Malawi	Singapore
Belgium	Fiji	Malaysia	South Africa
Benin	Finland	Mauritania	Spain
Bolivia	France	Mexico	Sri Lanka
Brazil	Gabon	Morocco	Sudan
Burkina Faso	Ghana	Netherlands	Suriname
Burundi	Greece	Nicaragua	Sweden
Cameroon	Guatemala	Niger	Thailand
Canada	Guyana	Nigeria	Togo
Central African Rep.	Honduras	Pakistan	Trinidad and Tobago
Chad	Hong Kong	Panama	Uganda
Chile	India	Papua New Guinea	United Kingdom
China	Ireland	Peru	United States
Colombia	Israel	Philippines	Uruguay
Congo, Dem. Rep.	Italy	Portugal	
Congo, Rep.	Ivory Coast	Rwanda	

Table A.2: Language ties: Common official and second languages (Krisztin and Fischer 2015)

English	**French**	**Spanish**	**Arabic**
Australia	Algeria	Bolivia	Algeria
Bahamas	Belgium	Chile	Chad
Cameroon	Benin	Colombia	Mauritania
Canada	Burkina Faso	Costa Rica	Morocco
Fiji	Burundi	Dominican Rep.	Sudan
Ghana	Cameroon	Ecuador	
Guyana	Canada	Guatemala	**Chinese**
India	Cent. African Rep.	Honduras	China
Ireland	Chad	Mexico	Hong Kong
Jamaica	Congo, Dem. Rep.	Nicaragua	Malaysia
Kenya	Congo, Rep.	Panama	Singapore
Malawi	France	Peru	
Nigeria	Gabon	Spain	**Malay**
Pakistan	Ivory Coast	Uruguay	Malaysia
Panama	Madagascar		Singapore
Papua New Guina	Morocco	**Dutch**	
Philippines	Niger	Belgium	
Rwanda	Rwanda	Netherlands	
Sierra Leone	Senegal	Suriname	
Singapore	Togo		
South Africa			
Sri Lanka	**Portugese**		
St. Kitts and Nevis	Brazil		
Suriname	Portugal		
Trinidad and Tobago			
Uganda			
United Kingdom			
USA			

Table A.3: Free trade and stronger forms of agreements in 2000 (Krisztin and Fischer 2015)

APTA	*CEMAC*	*EU*	Malaysia	*NAFTA*
India	Burundi	Austria	Mexico	Canada
Philippines	Cameroon	Belgium	Morocco	Mexico
Sri Lanka	Central African Rep.	Demark	Nicaragua	USA
	Chad	Finland	Pakistan	
ASEAN [AFTA]	Congo, Rep.	France	Peru	*PATCRA*
Malaysia	Congo. Dem. Rep.	Greece	Philippines	Australia
Philippines	Gabon	Ireland	Singapore	Papua New Guinea
Singapore		Italy	Sri Lanka	
Thailand	*COMESA*	Netherlands	Sudan	*SICA*
	Burundi	Portugal	Thailand	Costa Rica
CAN	Congo, Dem. Rep.	Spain	Trinidad and Tobago	Guatemala
Bolivia	Kenya	Sweden		Honduras
Colombia	Madagascar	United Kingdom	*LAIA*	Nicaragua
Ecuador	Rwanda	Uruguay	Bolivia	
Peru	Sudan		Brazil	*EU treaties*
	Uganda	*GSTP*	Chile	EU-Israel
CACM		Algeria	Colombia	EU-South Africa
Costa Rica	*ECOWAS*	Benin	Ecuador	
Guatemala	Benin	Bolivia	Mexico	*Bilateral treaties*
Honduras	Burkina Faso	Brazil	Panama	Canada-Chile
Nicaragua	Ghana	Cameroon	Peru	Canada-Israel
	Ivory Coast	Chile		Chile-Mexico
CARICOM	Niger	Colombia	*MERCOSUR*	Colombia-Mexico
Bahamas	Nigeria	Ecuador	Bolivia	Fiji-Papua New Guinea
Dominican Rep.	Senegal	Ghana	Brazil	Israel-Mexico
Guyana	Sierra Leone	Guyana	Chile	
Jamaica	Togo	India	Uruguay	
Saint Kitts and Nevis				
Suriname				
Trinidad and Tobago				

Note: Asia Pacific Trade Agreement (APTA), Asian Free Trade Area (AFTA), Andean Community (CAN), Central American Common Market (CACM), Caribbean Community and Common Market (CARICOM), Economic Community of Central African States (CEMAC), Common Market for Eastern and Southern Africa (COMESA), Economic Community of West African States (ECOWAS), European Free Trade Agreement (EFTA), Global System of Trade Preferences among Developing Countries (GSTP), Latin American Integration Association (LAIA), North American Free Trade Agreement (NAFTA), Mercado Comun del Sur (MERCOSUR), Agreement on Trade between Australia and New Guinea (PATCRA), Central American Integration System (SICA) (Source: WTO (2014))

Table A.4: Common currency ties

Euro:	Austria, Belgium, France, Finland, Ireland, Italy, Netherlands, Portual, Spain
US Dollar:	United States, Bahamas[1], Panama
West African CFA Franc[2,4]:	Benin, Burkina Faso, Ivory Coast, Niger, Senegal, Togo
Central African CFA Franc[3,4]:	Cameroon, Central African Republic, Chad, Republic of Congo, Gabon

Notes: 1) The Bahamian dollar is bagged to the US dollar on a one-to one basis. 2) CFA stands for African Financial Community. It is issued by the Central Bank of the West African States, located in Dakar, Senegal, for the countries of the West African Economic and Monetary Union. 3) CFA stands for Financial Cooperation in Central Africa. It is issued by the Bank of Central African States, located in Yaoundé, Cameroon, for the countries of the Economic and Monetary Union of Central Africa. 4) The two CFA Franc currencies, although theoretically separate, are effectively interchangeable.

Table A.5: Direct colonial ties

UNITED KINGDOM	Malawi	*FRANCE*	Morocco	Honduras
Australia	Malaysia	Algeria	Niger	Mexico
Bahamas	Nigeria	Benin	Senegal	Netherlands
Cameroon	Pakistan	Burkina Faso	Togo	Nicaragua
Fiji	Sierra Leone	Cameroon		Panama
Ghana	South Africa	Central African Rep.	*SPAIN*	Peru
Hong Kong	Sri Lanka	Chad	Bolivia	
India	St. Kitts amd Nevis	Congo, Dem. Rep.	Chile	*BELGIUM*
Ireland	Sudan	Congo, Rep.	Colombia	Congo, Dem. Rep.
Israel	Trinidad and Tobago	Gabon	Costa Rica	
Jamaica	Uganda	Madagascar	Ecuador	*PORTUGAL*
Kenya	United States	Mauritania	Guatemala	Brazil

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Table 1: Posterior means and standard deviations for ρ parameters

	Mean	Std. deviation	Mean/Std.	t-probability
SAR models				
Imports	0.6709	0.1550	4.3288	0.0000
Exports	0.6653	0.1550	4.2928	0.0001
SDM models				
Imports	0.6035	0.1927	3.1313	0.0025
Exports	0.6021	0.1927	3.1241	0.0026

Table 2: Counts of SAR and SDM γ estimates that are different from zero

	SAR models		SDM models	
	# Imports $\neq$ zero	# Exports $\neq$ zero	# Imports $\neq$ zero	# Exports $\neq$ zero
Wspace	66	65	58	59
Wcurrency	27	26	18	18
Wlanguage	41	18	38	14
Wtrade	42	46	37	41
Wcolonial	52	58	52	61

Table 3: Means and standard deviations of SAR and SDM γ estimates across 74 countries

	SAR models				SDM models			
	Imports		Exports		Imports		Exports	
	Mean	σ_γ	Mean	σ_γ	Mean	σ_γ	Mean	σ_γ
Wspace	0.3642	0.1800	0.3375	0.1936	0.3500	0.2163	0.3279	0.2163
Wcurrency	0.1340	0.1567	0.1241	0.1579	0.1117	0.1535	0.1144	0.1535
Wlanguage	0.1310	0.0976	0.0737	0.1067	0.1411	0.1399	0.0695	0.1399
Wtrade	0.2237	0.2051	0.2333	0.1838	0.2335	0.2301	0.2244	0.2301
Wcolonial	0.1471	0.0911	0.2314	0.1339	0.1637	0.0985	0.2638	0.0985

Table 4: SDM model γ estimates for Wspace import and export flow models

	Import models				Export models		
Country	Lower 0.05	Mean	Upper 0.95	Country	Lower 0.05	Mean	Upper 0.95
United Kingdom	0.0000	0.0120	0.0888	Cameroon	0.0000	0.0005	0.0013
Portugal	0.0000	0.0130	0.0916	Sierra Leone	0.0000	0.0026	0.0163
Bolivia	0.0000	0.0248	0.1304	Benin	0.0000	0.0097	0.0603
Sierra Leone	0.0000	0.0273	0.1592	Sudan	0.0000	0.0207	0.1291
Canada	0.0000	0.0505	0.1918	Niger	0.0000	0.0230	0.0969
St. Kitts and Nevis	0.0000	0.0512	0.1600	Rwanda	0.0000	0.0289	0.1377
Jamaica	0.0000	0.0561	0.1722	Canada	0.0000	0.0327	0.1978
Algeria	0.0000	0.0570	0.1731	Ecuador	0.0000	0.0714	0.2015
Panama	0.0000	0.0651	0.1716	Ivory Coast	0.0260	0.1093	0.1960
Guyana	0.0000	0.0747	0.2789	Ghana	0.0217	0.1148	0.2046
Israel	0.0000	0.0879	0.2268	Burkina Faso	0.0176	0.1159	0.2048
Central African Republic	0.0000	0.1013	0.2716	Australia	0.0007	0.1188	0.2357
Sri Lanka	0.0032	0.1022	0.2076	Uruguay	0.0002	0.1195	0.2551
Morocco	0.0011	0.1336	0.3091	Kenya	0.0013	0.1233	0.2872
Chad	0.0101	0.1516	0.2893	Trinidad and Tobago	0.0462	0.1366	0.2302
Netherlands	0.0028	0.1756	0.3495	Jamaica	0.0267	0.1461	0.2646
Guatemala	0.0893	0.1874	0.2774	Chad	0.0289	0.1501	0.2571
Trinidad and Tobago	0.1095	0.1887	0.2958	St. Kitts and Nevis	0.0618	0.1707	0.2796
Australia	0.1384	0.2178	0.3075	Portugal	0.0020	0.1764	0.3185
Austria	0.0016	0.2227	0.4871	Gabon	0.0793	0.1775	0.2726
Cameroon	0.1717	0.2300	0.2911	Madagascar	0.0506	0.1825	0.3354
Malaysia	0.1511	0.2318	0.3285	Spain	0.0001	0.1826	0.4578
United States	0.1733	0.2330	0.2987	Senegal	0.0735	0.1827	0.2840
Costa Rica	0.1877	0.2417	0.3007	Congo, Rep.	0.1014	0.1921	0.2899
Malawi	0.1792	0.2565	0.3354	Peru	0.0865	0.1950	0.3100
Pakistan	0.1197	0.2583	0.4094	Nicaragua	0.1298	0.2187	0.3097
Madagascar	0.1534	0.2619	0.3687	Nigeria	0.1479	0.2380	0.3212
Italy	0.0743	0.2652	0.4728	Togo	0.1438	0.2394	0.3291
Benin	0.1789	0.2663	0.3666	Bahamas, The	0.0811	0.2407	0.4035
Congo, Rep.	0.1835	0.2799	0.3785	Ireland	0.0021	0.2454	0.4903
Burundi	0.1010	0.2875	0.4947	Dominican Republic	0.1725	0.2572	0.3499
Denmark	0.0884	0.3020	0.5012	Malawi	0.1935	0.2757	0.3659
Uganda	0.1080	0.3048	0.5254	Mauritania	0.1453	0.2768	0.4053
Ghana	0.2194	0.3058	0.4015	Chile	0.1615	0.2846	0.4019
Bahamas, The	0.1739	0.3126	0.4472	Morocco	0.1242	0.2855	0.4431
Mexico	0.2407	0.3176	0.4050	United States	0.0216	0.2980	0.5595
Singapore	0.1829	0.3283	0.4686	Algeria	0.1352	0.2980	0.4665
Ecuador	0.2479	0.3454	0.4516	Panama	0.2334	0.3044	0.3776
Peru	0.2441	0.3477	0.4544	India	0.2432	0.3165	0.4107
South Africa	0.2746	0.3497	0.4160	Papua New Guinea	0.1940	0.3174	0.4487
Dominican Republic	0.2369	0.3498	0.4803	South Africa	0.2387	0.3203	0.4140
Niger	0.2168	0.3585	0.4932	Fiji	0.2028	0.3223	0.4479
Hong Kong SAR, China	0.2684	0.3587	0.4539	Austria	0.0002	0.3299	0.7408
Philippines	0.2498	0.3649	0.4862	China	0.2270	0.3426	0.4507
Mauritania	0.2182	0.3739	0.5275	Brazil	0.1946	0.3454	0.4895
Nicaragua	0.3004	0.3820	0.4713	France	0.2605	0.3548	0.4515
Colombia	0.2699	0.3866	0.5049	Bolivia	0.1472	0.3654	0.5956
Sweden	0.2366	0.3879	0.5375	Honduras	0.2768	0.3699	0.4613
Uruguay	0.2728	0.3940	0.5200	Guatemala	0.2865	0.3702	0.4566
Honduras	0.3042	0.4004	0.5147	Israel	0.2416	0.3866	0.5221
France	0.2833	0.4371	0.5743	Guyana	0.2935	0.3985	0.5046
India	0.3573	0.4438	0.5143	Central African Republic	0.1611	0.4020	0.6595
Ivory Coast	0.4196	0.4623	0.5125	Burundi	0.1852	0.4028	0.6377
China	0.3996	0.4731	0.5429	Costa Rica	0.3254	0.4062	0.4939
Sudan	0.2156	0.4766	0.7861	Mexico	0.3347	0.4099	0.4893
Kenya	0.3762	0.4776	0.5683	Colombia	0.3564	0.4398	0.5192
Senegal	0.4298	0.4831	0.5324	Malaysia	0.3158	0.4468	0.5618
Nigeria	0.3914	0.5024	0.6042	Uganda	0.3252	0.4821	0.6695
Spain	0.2879	0.5048	0.7143	Denmark	0.3774	0.4867	0.5802
Fiji	0.3168	0.5067	0.7411	Thailand	0.3824	0.4884	0.6023
Chile	0.4258	0.5297	0.6105	Netherlands	0.3086	0.5185	0.7656
Burkina Faso	0.4338	0.5562	0.7127	Sri Lanka	0.3652	0.5214	0.6852
Rwanda	0.3095	0.5687	0.9458	Philippines	0.3503	0.5401	0.7176
Thailand	0.4596	0.5858	0.7017	United Kingdom	0.2709	0.5405	0.9588
Belgium	0.4114	0.5925	0.8195	Hong Kong SAR, China	0.3874	0.5554	0.7136
Greece	0.4591	0.6331	0.8420	Congo, Dem. Rep.	0.4839	0.6079	0.7411
Gabon	0.5213	0.6526	0.7880	Greece	0.3638	0.6147	0.9175
Ireland	0.5575	0.6701	0.8003	Italy	0.2442	0.7137	0.9993
Papua New Guinea	0.5604	0.7424	1.0000	Finland	0.5178	0.7219	0.9772
Suriname	0.5551	0.7974	0.9988	Suriname	0.4777	0.7633	0.9999
Congo, Dem. Rep.	0.5535	0.8010	1.0000	Sweden	0.5348	0.7729	0.9981
Finland	0.5953	0.8227	1.0000	Pakistan	0.5528	0.8281	1.0000
Brazil	0.6143	0.8265	0.9998	Singapore	0.5555	0.8444	1.0000
Togo	0.6539	0.8693	1.0000	Belgium	0.8494	0.9735	1.0000

Table 5: SDM model γ estimates for Wcurrency import and export flow models

	Import models				Export models		
Country	Lower 0.05	Mean	Upper 0.95	Country	Lower 0.05	Mean	Upper 0.95
United Kingdom	0.0000	0.0001	0.0001	Cameroon	0.0000	0.0002	0.0002
Portugal	0.0000	0.0001	0.0001	Sierra Leone	0.0000	0.0002	0.0003
Bolivia	0.0000	0.0002	0.0002	Benin	0.0000	0.0003	0.0003
Sierra Leone	0.0000	0.0002	0.0006	Sudan	0.0000	0.0003	0.0001
Canada	0.0000	0.0003	0.0010	Niger	0.0000	0.0004	0.0006
St. Kitts and Nevis	0.0000	0.0003	0.0003	Rwanda	0.0000	0.0004	0.0013
Jamaica	0.0000	0.0003	0.0005	Canada	0.0000	0.0005	0.0006
Algeria	0.0000	0.0004	0.0002	Ecuador	0.0000	0.0006	0.0011
Panama	0.0000	0.0004	0.0010	Ivory Coast	0.0000	0.0006	0.0018
Guyana	0.0000	0.0004	0.0017	Ghana	0.0000	0.0006	0.0016
Israel	0.0000	0.0005	0.0026	Burkina Faso	0.0000	0.0007	0.0024
Central African Republic	0.0000	0.0005	0.0013	Australia	0.0000	0.0009	0.0038
Sri Lanka	0.0000	0.0005	0.0015	Uruguay	0.0000	0.0009	0.0024
Morocco	0.0000	0.0006	0.0013	Kenya	0.0000	0.0009	0.0031
Chad	0.0000	0.0006	0.0029	Trinidad and Tobago	0.0000	0.0011	0.0030
Netherlands	0.0000	0.0006	0.0011	Jamaica	0.0000	0.0013	0.0064
Guatemala	0.0000	0.0010	0.0031	Chad	0.0000	0.0014	0.0063
Trinidad and Tobago	0.0000	0.0013	0.0070	St. Kitts and Nevis	0.0000	0.0025	0.0143
Australia	0.0000	0.0014	0.0081	Portugal	0.0000	0.0026	0.0147
Austria	0.0000	0.0014	0.0068	Gabon	0.0000	0.0027	0.0185
Cameroon	0.0000	0.0020	0.0101	Madagascar	0.0000	0.0033	0.0225
Malaysia	0.0000	0.0022	0.0135	Spain	0.0000	0.0038	0.0199
United States	0.0000	0.0023	0.0107	Senegal	0.0000	0.0038	0.0144
Costa Rica	0.0000	0.0025	0.0101	Congo, Rep.	0.0000	0.0041	0.0291
Malawi	0.0000	0.0030	0.0215	Peru	0.0000	0.0061	0.0399
Pakistan	0.0000	0.0032	0.0219	Nicaragua	0.0000	0.0083	0.0601
Madagascar	0.0000	0.0035	0.0250	Nigeria	0.0000	0.0092	0.0619
Italy	0.0000	0.0040	0.0241	Togo	0.0000	0.0100	0.0671
Benin	0.0000	0.0048	0.0269	Bahamas	0.0000	0.0130	0.0891
Congo, Rep.	0.0000	0.0049	0.0337	Ireland	0.0000	0.0180	0.1084
Burundi	0.0000	0.0064	0.0458	Dominican Republic	0.0000	0.0190	0.1137
Denmark	0.0000	0.0104	0.0686	Malawi	0.0000	0.0192	0.0987
Uganda	0.0000	0.0124	0.0769	Mauritania	0.0000	0.0205	0.1188
Ghana	0.0000	0.0158	0.1087	Chile	0.0000	0.0223	0.1358
Bahamas	0.0000	0.0162	0.0958	Morocco	0.0000	0.0233	0.0931
Mexico	0.0000	0.0170	0.1055	United States	0.0000	0.0235	0.1415
Singapore	0.0000	0.0189	0.1002	Algeria	0.0000	0.0257	0.1452
Ecuador	0.0000	0.0293	0.1560	Panama	0.0000	0.0262	0.1584
Peru	0.0000	0.0343	0.0960	India	0.0000	0.0272	0.1470
South Africa	0.0000	0.0388	0.1917	Papua New Guinea	0.0000	0.0288	0.1399
Dominican Republic	0.0000	0.0396	0.1588	South Africa	0.0000	0.0294	0.1636
Niger	0.0000	0.0501	0.1767	Fiji	0.0000	0.0358	0.1770
Hong Kong	0.0000	0.0527	0.2146	Austria	0.0000	0.0564	0.1614
Philippines	0.0000	0.0603	0.2413	China	0.0000	0.0609	0.2310
Mauritania	0.0000	0.0676	0.2437	Brazil	0.0000	0.0617	0.1733
Nicaragua	0.0000	0.0748	0.3530	France	0.0000	0.0661	0.1850
Colombia	0.0000	0.0824	0.2337	Bolivia	0.0000	0.0684	0.2227
Sweden	0.0001	0.0838	0.1832	Honduras	0.0000	0.0731	0.2573
Uruguay	0.0000	0.0906	0.2755	Guatemala	0.0000	0.0781	0.3508
Honduras	0.0000	0.1092	0.2260	Israel	0.0000	0.0798	0.2134
France	0.0000	0.1120	0.3153	Guyana	0.0000	0.0889	0.2093
India	0.0080	0.1158	0.2257	Central African Republic	0.0000	0.1213	0.3150
Ivory Coast	0.0000	0.1326	0.3171	Burundi	0.0015	0.1301	0.2621
China	0.0000	0.1631	0.2932	Costa Rica	0.0000	0.1596	0.4616
Sudan	0.0004	0.1701	0.3259	Mexico	0.0000	0.1647	0.3973
Kenya	0.1450	0.1915	0.2374	Colombia	0.0438	0.1713	0.2818
Senegal	0.0001	0.1948	0.4361	Malaysia	0.1056	0.1732	0.2499
Nigeria	0.0632	0.2177	0.3444	Uganda	0.0039	0.1792	0.3118
Spain	0.1516	0.2572	0.3523	Denmark	0.1348	0.2151	0.2895
Fiji	0.1486	0.2645	0.3870	Thailand	0.0337	0.2165	0.3612
Chile	0.2090	0.2671	0.3312	Netherlands	0.0621	0.2331	0.3859
Burkina Faso	0.0711	0.3175	0.4970	Sri Lanka	0.2177	0.3043	0.3920
Rwanda	0.2270	0.3199	0.3978	Philippines	0.2421	0.3052	0.3713
Thailand	0.1413	0.3248	0.5198	United Kingdom	0.2873	0.3497	0.4109
Belgium	0.2200	0.3268	0.4368	Hong Kong	0.2502	0.3559	0.4520
Greece	0.3077	0.3743	0.4408	Congo, Dem. Rep.	0.2956	0.3634	0.4291
Gabon	0.2945	0.3866	0.4736	Greece	0.2931	0.4131	0.5347
Ireland	0.2096	0.3885	0.5709	Italy	0.3583	0.4184	0.4761
Papua New Guinea	0.3318	0.4289	0.5119	Finland	0.3723	0.4568	0.5356
Suriname	0.3754	0.4348	0.4984	Suriname	0.3638	0.4597	0.5615
Congo, Dem. Rep.	0.2895	0.4624	0.6055	Sweden	0.3914	0.4636	0.5424
Finland	0.3490	0.4730	0.5935	Pakistan	0.4323	0.5350	0.6260
Brazil	0.3354	0.4856	0.6258	Singapore	0.4395	0.6153	0.7784
Togo	0.4176	0.5031	0.5980	Belgium	0.4640	0.6257	0.8206

Table 6: SDM model γ estimates for Wlanguage import and export flow models

	Import models				Export models		
Country	Lower 0.05	Mean	Upper 0.95	Country	Lower 0.05	Mean	Upper 0.95
United Kingdom	0.0000	0.0005	0.0008	Cameroon	0.0000	0.0001	0.0000
Portugal	0.0000	0.0006	0.0016	Sierra Leone	0.0000	0.0002	0.0002
Bolivia	0.0000	0.0013	0.0071	Benin	0.0000	0.0003	0.0005
Sierra Leone	0.0000	0.0022	0.0124	Sudan	0.0000	0.0004	0.0004
Canada	0.0000	0.0024	0.0134	Niger	0.0000	0.0004	0.0005
St. Kitts and Nevis	0.0000	0.0032	0.0200	Rwanda	0.0000	0.0004	0.0009
Jamaica	0.0000	0.0034	0.0257	Canada	0.0000	0.0006	0.0013
Algeria	0.0000	0.0035	0.0205	Ecuador	0.0000	0.0006	0.0016
Panama	0.0000	0.0036	0.0261	Ivory Coast	0.0000	0.0008	0.0032
Guyana	0.0000	0.0066	0.0444	Ghana	0.0000	0.0009	0.0012
Israel	0.0000	0.0082	0.0514	Burkina Faso	0.0000	0.0011	0.0030
Central African Republic	0.0000	0.0167	0.0959	Australia	0.0000	0.0011	0.0031
Sri Lanka	0.0000	0.0199	0.0896	Uruguay	0.0000	0.0012	0.0046
Morocco	0.0000	0.0209	0.1176	Kenya	0.0000	0.0014	0.0074
Chad	0.0000	0.0229	0.0972	Trinidad and Tobago	0.0000	0.0015	0.0083
Netherlands	0.0000	0.0294	0.1281	Jamaica	0.0000	0.0016	0.0114
Guatemala	0.0000	0.0313	0.1109	Chad	0.0000	0.0018	0.0110
Trinidad and Tobago	0.0000	0.0417	0.1322	St. Kitts and Nevis	0.0000	0.0024	0.0147
Australia	0.0000	0.0475	0.1625	Portugal	0.0000	0.0025	0.0147
Austria	0.0000	0.0533	0.1715	Gabon	0.0000	0.0027	0.0182
Cameroon	0.0000	0.0561	0.2108	Madagascar	0.0000	0.0028	0.0160
Malaysia	0.0000	0.0575	0.1709	Spain	0.0000	0.0035	0.0237
United States	0.0000	0.0577	0.2333	Senegal	0.0000	0.0035	0.0232
Costa Rica	0.0000	0.0629	0.2490	Congo, Rep.	0.0000	0.0039	0.0256
Malawi	0.0000	0.0633	0.2122	Peru	0.0000	0.0056	0.0360
Pakistan	0.0000	0.0661	0.2038	Nicaragua	0.0000	0.0063	0.0416
Madagascar	0.0000	0.0669	0.1726	Nigeria	0.0000	0.0065	0.0442
Italy	0.0012	0.0836	0.1712	Togo	0.0000	0.0071	0.0428
Benin	0.0001	0.0849	0.1832	Bahamas	0.0000	0.0078	0.0515
Congo, Rep.	0.0366	0.0880	0.1446	Ireland	0.0000	0.0078	0.0528
Burundi	0.0007	0.0915	0.1828	Dominican Republic	0.0000	0.0083	0.0530
Denmark	0.0035	0.0917	0.1863	Malawi	0.0000	0.0093	0.0562
Uganda	0.0217	0.0961	0.1753	Mauritania	0.0000	0.0099	0.0625
Ghana	0.0108	0.0967	0.1819	Chile	0.0000	0.0106	0.0619
Bahamas	0.0000	0.0969	0.3879	Morocco	0.0000	0.0123	0.0724
Mexico	0.0000	0.1014	0.2417	United States	0.0000	0.0136	0.0678
Singapore	0.0309	0.1021	0.1831	Algeria	0.0000	0.0148	0.0788
Ecuador	0.0004	0.1080	0.2491	Panama	0.0000	0.0174	0.0978
Peru	0.0416	0.1137	0.1964	India	0.0000	0.0179	0.0812
South Africa	0.0416	0.1138	0.1965	Papua New Guinea	0.0000	0.0179	0.1109
Dominican Republic	0.0001	0.1170	0.2915	South Africa	0.0000	0.0236	0.1150
Niger	0.0492	0.1208	0.2037	Fiji	0.0000	0.0256	0.1107
Hong Kong	0.0260	0.1288	0.2397	Austria	0.0000	0.0303	0.1212
Philippines	0.0694	0.1319	0.2020	China	0.0000	0.0314	0.1234
Mauritania	0.0009	0.1337	0.3059	Brazil	0.0000	0.0323	0.1466
Nicaragua	0.0747	0.1355	0.2024	France	0.0000	0.0346	0.1198
Colombia	0.0151	0.1375	0.2284	Bolivia	0.0000	0.0491	0.1511
Sweden	0.0703	0.1545	0.2353	Honduras	0.0000	0.0530	0.1810
Uruguay	0.0662	0.1704	0.2662	Guatemala	0.0001	0.0609	0.1585
Honduras	0.0639	0.1717	0.2826	Israel	0.0000	0.0653	0.1616
France	0.1428	0.1838	0.2292	Guyana	0.0000	0.0698	0.2033
India	0.1090	0.1899	0.2793	Central African Republic	0.0000	0.0740	0.1630
Ivory Coast	0.1239	0.1980	0.2718	Burundi	0.0000	0.0755	0.2050
China	0.1437	0.2013	0.2640	Costa Rica	0.0000	0.0813	0.2098
Sudan	0.1275	0.2066	0.2857	Mexico	0.0000	0.0866	0.2615
Kenya	0.0870	0.2102	0.3330	Colombia	0.0000	0.0940	0.2577
Senegal	0.1705	0.2187	0.2638	Malaysia	0.0005	0.0985	0.2098
Nigeria	0.0591	0.2193	0.3611	Uganda	0.0000	0.1016	0.3119
Spain	0.1629	0.2215	0.2843	Denmark	0.0438	0.1028	0.1660
Fiji	0.0917	0.2239	0.3689	Thailand	0.0241	0.1044	0.1869
Chile	0.1575	0.2400	0.3296	Netherlands	0.0020	0.1059	0.2254
Burkina Faso	0.1618	0.2450	0.3356	Sri Lanka	0.0079	0.1096	0.2211
Rwanda	0.1433	0.2454	0.3527	Philippines	0.0253	0.1187	0.2108
Thailand	0.1621	0.2575	0.3439	United Kingdom	0.0113	0.1223	0.2394
Belgium	0.1449	0.2582	0.3692	Hong Kong	0.0384	0.1342	0.2332
Greece	0.1510	0.2732	0.4082	Congo, Dem. Rep.	0.0488	0.1366	0.2224
Gabon	0.0786	0.2832	0.4442	Greece	0.0820	0.1505	0.2290
Ireland	0.1822	0.3008	0.4243	Italy	0.0920	0.1940	0.3032
Papua New Guinea	0.0988	0.3130	0.5123	Finland	0.1176	0.2229	0.3345
Suriname	0.2312	0.3778	0.5295	Suriname	0.1405	0.2238	0.2986
Congo, Dem. Rep.	0.2230	0.4065	0.5979	Sweden	0.1711	0.2871	0.4146
Finland	0.2664	0.4123	0.5627	Pakistan	0.0364	0.4564	0.9153
Brazil	0.2688	0.4632	0.6613	Singapore	0.2105	0.4817	0.7892
Togo	0.5818	0.8732	1.0000	Belgium	0.6401	0.8971	1.0000

Table 7: SDM model γ estimates for Wtrade import and export flow models

	Import models				Export models		
Country	Lower 0.05	Mean	Upper 0.95	Country	Lower 0.05	Mean	Upper 0.95
United Kingdom	0.0000	0.0010	0.0024	Cameroon	0.0000	0.0007	0.0014
Portugal	0.0000	0.0013	0.0079	Sierra Leone	0.0000	0.0015	0.0104
Bolivia	0.0000	0.0027	0.0200	Benin	0.0000	0.0074	0.0512
Sierra Leone	0.0000	0.0035	0.0201	Sudan	0.0000	0.0083	0.0562
Canada	0.0000	0.0045	0.0300	Niger	0.0000	0.0084	0.0602
St. Kitts and Nevis	0.0000	0.0050	0.0387	Rwanda	0.0000	0.0119	0.0737
Jamaica	0.0000	0.0056	0.0413	Canada	0.0000	0.0130	0.0841
Algeria	0.0000	0.0060	0.0379	Ecuador	0.0000	0.0132	0.0759
Panama	0.0000	0.0060	0.0434	Ivory Coast	0.0000	0.0156	0.1089
Guyana	0.0000	0.0067	0.0444	Ghana	0.0000	0.0186	0.1143
Israel	0.0000	0.0133	0.0824	Burkina Faso	0.0000	0.0199	0.1277
Central African Republic	0.0000	0.0135	0.0784	Australia	0.0000	0.0206	0.1284
Sri Lanka	0.0000	0.0158	0.1092	Uruguay	0.0000	0.0211	0.1037
Morocco	0.0000	0.0173	0.1029	Kenya	0.0000	0.0226	0.1033
Chad	0.0000	0.0185	0.1187	Trinidad and Tobago	0.0000	0.0284	0.1482
Netherlands	0.0000	0.0190	0.1199	Jamaica	0.0000	0.0398	0.1550
Guatemala	0.0000	0.0213	0.1166	Chad	0.0000	0.0426	0.2139
Trinidad and Tobago	0.0000	0.0237	0.1376	St. Kitts and Nevis	0.0000	0.0435	0.2305
Australia	0.0000	0.0273	0.1563	Portugal	0.0000	0.0455	0.2201
Austria	0.0000	0.0280	0.1889	Gabon	0.0000	0.0497	0.2672
Cameroon	0.0000	0.0289	0.1215	Madagascar	0.0000	0.0511	0.2397
Malaysia	0.0000	0.0321	0.1389	Spain	0.0000	0.0552	0.1610
United States	0.0000	0.0429	0.1305	Senegal	0.0000	0.0568	0.1694
Costa Rica	0.0000	0.0513	0.1513	Congo, Rep.	0.0000	0.0732	0.2427
Malawi	0.0000	0.0533	0.2143	Peru	0.0000	0.0746	0.2924
Pakistan	0.0000	0.0593	0.2040	Nicaragua	0.0000	0.0803	0.2636
Madagascar	0.0000	0.0878	0.2296	Nigeria	0.0000	0.0895	0.2132
Italy	0.0010	0.0916	0.2000	Togo	0.0000	0.0918	0.2356
Benin	0.0001	0.0954	0.2395	Bahamas	0.0000	0.1030	0.2750
Congo, Rep.	0.0004	0.0999	0.2032	Ireland	0.0003	0.1231	0.2791
Burundi	0.0000	0.1032	0.2387	Dominican Republic	0.0214	0.1391	0.2529
Denmark	0.0001	0.1197	0.2460	Malawi	0.0013	0.1448	0.3007
Uganda	0.0026	0.1236	0.2293	Mauritania	0.0582	0.1482	0.2561
Ghana	0.0006	0.1259	0.2702	Chile	0.1086	0.1865	0.2742
Bahamas	0.0000	0.1414	0.3131	Morocco	0.0291	0.1870	0.3322
Mexico	0.0389	0.1601	0.2773	United States	0.1049	0.1891	0.2805
Singapore	0.0989	0.1641	0.2377	Algeria	0.0860	0.2127	0.3510
Ecuador	0.0533	0.1805	0.2902	Panama	0.1068	0.2176	0.3350
Peru	0.0743	0.1853	0.3006	India	0.0989	0.2212	0.3441
South Africa	0.0877	0.1967	0.3012	Papua New Guinea	0.1623	0.2224	0.2877
Dominican Republic	0.1634	0.2306	0.3130	South Africa	0.1178	0.2286	0.3503
Niger	0.1000	0.2416	0.3912	Fiji	0.1281	0.2320	0.3222
Hong Kong	0.0039	0.2419	0.4463	Austria	0.0046	0.2349	0.4812
Philippines	0.0940	0.2507	0.3989	China	0.1515	0.2389	0.3348
Mauritania	0.0000	0.2531	0.5033	Brazil	0.1455	0.2428	0.3352
Nicaragua	0.0194	0.2752	0.4964	France	0.0685	0.2517	0.4185
Colombia	0.1423	0.2753	0.4169	Bolivia	0.1765	0.2553	0.3326
Sweden	0.0665	0.2994	0.5228	Honduras	0.1743	0.2607	0.3394
Uruguay	0.2299	0.2998	0.3728	Guatemala	0.1724	0.2743	0.3777
Honduras	0.1884	0.3002	0.4239	Israel	0.2015	0.2867	0.3651
France	0.2059	0.3167	0.4313	Guyana	0.0424	0.2893	0.5269
India	0.2493	0.3202	0.4001	Central African Republic	0.0000	0.2973	0.5718
Ivory Coast	0.2443	0.3419	0.4231	Burundi	0.1613	0.3047	0.4557
China	0.2672	0.3427	0.4246	Costa Rica	0.1984	0.3184	0.4236
Sudan	0.2435	0.3640	0.4788	Mexico	0.1522	0.3310	0.5111
Kenya	0.2313	0.3652	0.4803	Colombia	0.2596	0.3351	0.4096
Senegal	0.2773	0.3958	0.5115	Malaysia	0.2726	0.3554	0.4373
Nigeria	0.3329	0.3960	0.4688	Uganda	0.1319	0.3585	0.5847
Spain	0.3237	0.4146	0.4966	Denmark	0.2869	0.3921	0.4901
Fiji	0.3354	0.4223	0.5291	Thailand	0.2112	0.3979	0.6006
Chile	0.3741	0.4317	0.4848	Netherlands	0.3223	0.3982	0.4922
Burkina Faso	0.2810	0.4546	0.6468	Sri Lanka	0.2711	0.4024	0.5456
Rwanda	0.3643	0.4613	0.5458	Philippines	0.3538	0.4351	0.5114
Thailand	0.2119	0.4638	0.7407	United Kingdom	0.3165	0.4433	0.5683
Belgium	0.3798	0.4698	0.5620	Hong Kong	0.3060	0.4634	0.6036
Greece	0.0572	0.5037	0.9149	Congo, Dem. Rep.	0.2871	0.5481	0.8235
Gabon	0.4647	0.5627	0.6568	Greece	0.4782	0.5497	0.6307
Ireland	0.5006	0.6364	0.7835	Italy	0.4552	0.5523	0.6441
Papua New Guinea	0.5696	0.6768	0.7817	Finland	0.4747	0.5790	0.6897
Suriname	0.5663	0.6852	0.7738	Suriname	0.4409	0.5905	0.7613
Congo, Dem. Rep.	0.6131	0.7449	0.8630	Sweden	0.5202	0.6058	0.6896
Finland	0.6464	0.7500	0.8423	Pakistan	0.5553	0.6458	0.7389
Brazil	0.5468	0.8081	0.9994	Singapore	0.5541	0.6846	0.8197
Togo	0.6617	0.9022	1.0000	Belgium	0.3740	0.7247	0.9995

Table 8: SDM model γ estimates for W colonial ties import and export flow models

	Import models				Export models		
Country	Lower 0.05	Mean	Upper 0.95	Country	Lower 0.05	Mean	Upper 0.95
United Kingdom	0.0000	0.0226	0.0677	Cameroon	0.0000	0.0211	0.1258
Portugal	0.0024	0.0269	0.0723	Sierra Leone	0.0027	0.0421	0.1096
Bolivia	0.0017	0.0287	0.0768	Benin	0.0000	0.0465	0.1746
Sierra Leone	0.0029	0.0310	0.0849	Sudan	0.0042	0.0502	0.1255
Canada	0.0033	0.0444	0.1147	Niger	0.0018	0.0581	0.1539
St. Kitts and Nevis	0.0021	0.0473	0.1206	Rwanda	0.0001	0.0624	0.1838
Jamaica	0.0035	0.0502	0.1206	Canada	0.0070	0.0710	0.1527
Algeria	0.0000	0.0586	0.1634	Ecuador	0.0000	0.0835	0.2360
Panama	0.0051	0.0604	0.1492	Ivory Coast	0.0003	0.0938	0.2592
Guyana	0.0042	0.0608	0.1508	Ghana	0.0165	0.0946	0.1762
Israel	0.0037	0.0654	0.1759	Burkina Faso	0.0000	0.0967	0.2580
Central African Republic	0.0038	0.0662	0.1853	Australia	0.0165	0.0983	0.1873
Sri Lanka	0.0000	0.0668	0.1840	Uruguay	0.0162	0.1147	0.2463
Morocco	0.0049	0.0691	0.1696	Kenya	0.0094	0.1175	0.2712
Chad	0.0000	0.0711	0.2734	Trinidad and Tobago	0.0297	0.1288	0.2264
Netherlands	0.0103	0.0759	0.1611	Jamaica	0.0251	0.1291	0.2348
Guatemala	0.0000	0.0766	0.2818	Chad	0.0012	0.1372	0.3081
Trinidad and Tobago	0.0074	0.0777	0.1853	St. Kitts and Nevis	0.0512	0.1387	0.2240
Australia	0.0174	0.0797	0.1542	Portugal	0.0005	0.1507	0.4358
Austria	0.0117	0.0827	0.1785	Gabon	0.0270	0.1628	0.2738
Cameroon	0.0152	0.0885	0.1831	Madagascar	0.0458	0.1630	0.2754
Malaysia	0.0000	0.0914	0.2479	Spain	0.0113	0.1690	0.4415
United States	0.0122	0.0937	0.1863	Senegal	0.0432	0.1742	0.2920
Costa Rica	0.0079	0.0948	0.2308	Congo, Rep.	0.0281	0.1789	0.3261
Malawi	0.0122	0.0950	0.2162	Peru	0.0129	0.1946	0.3534
Pakistan	0.0142	0.1023	0.2185	Nicaragua	0.0590	0.1951	0.3153
Madagascar	0.0441	0.1084	0.1676	Nigeria	0.0979	0.1972	0.2874
Italy	0.0005	0.1124	0.2657	Togo	0.1044	0.2035	0.2862
Benin	0.0612	0.1180	0.1771	Bahamas	0.0953	0.2059	0.3003
Congo, Rep.	0.0320	0.1198	0.2036	Ireland	0.0240	0.2084	0.4509
Burundi	0.0009	0.1204	0.3113	Dominican Republic	0.1437	0.2280	0.3034
Denmark	0.0002	0.1247	0.2718	Malawi	0.1609	0.2456	0.3246
Uganda	0.0487	0.1251	0.2001	Mauritania	0.1229	0.2465	0.3585
Ghana	0.0227	0.1277	0.2478	Chile	0.1689	0.2482	0.3204
Bahamas	0.0325	0.1301	0.2321	Morocco	0.1742	0.2559	0.3309
Mexico	0.0344	0.1320	0.2147	United States	0.0646	0.2575	0.4257
Singapore	0.0444	0.1346	0.2169	Algeria	0.0573	0.2640	0.4521
Ecuador	0.0306	0.1363	0.2558	Panama	0.1146	0.2711	0.4168
Peru	0.0279	0.1508	0.2792	India	0.2033	0.2714	0.3306
South Africa	0.0161	0.1532	0.3452	Papua New Guinea	0.2148	0.2781	0.3413
Dominican Republic	0.0432	0.1588	0.2635	South Africa	0.2048	0.2807	0.3507
Niger	0.0217	0.1597	0.3459	Fiji	0.1181	0.2817	0.4317
Hong Kong	0.0261	0.1681	0.3372	Austria	0.1414	0.2818	0.4041
Philippines	0.0571	0.1839	0.3042	China	0.1202	0.2822	0.4396
Mauritania	0.0419	0.1934	0.3410	Brazil	0.2097	0.2921	0.3567
Nicaragua	0.1207	0.1937	0.2623	France	0.2408	0.3143	0.3872
Colombia	0.0950	0.1949	0.2706	Bolivia	0.2247	0.3212	0.4050
Sweden	0.1102	0.1985	0.2764	Honduras	0.2344	0.3244	0.4164
Uruguay	0.0251	0.1997	0.3990	Guatemala	0.1735	0.3355	0.4756
Honduras	0.0658	0.2046	0.3122	Israel	0.0560	0.3374	0.5803
France	0.0982	0.2090	0.3121	Guyana	0.2225	0.3433	0.4600
India	0.1533	0.2146	0.2717	Central African Republic	0.2372	0.3495	0.4468
Ivory Coast	0.1187	0.2172	0.3124	Burundi	0.2319	0.3507	0.4554
China	0.1292	0.2190	0.2993	Costa Rica	0.2592	0.3523	0.4361
Sudan	0.1092	0.2203	0.3094	Mexico	0.2584	0.3674	0.4618
Kenya	0.0412	0.2333	0.4183	Colombia	0.2093	0.3680	0.5097
Senegal	0.1529	0.2373	0.3128	Malaysia	0.2945	0.3699	0.4411
Nigeria	0.1255	0.2429	0.3503	Uganda	0.3014	0.3807	0.4485
Spain	0.1880	0.2451	0.2970	Denmark	0.3025	0.3826	0.4561
Fiji	0.1388	0.2505	0.3541	Thailand	0.2494	0.3871	0.5118
Chile	0.1922	0.2527	0.3084	Netherlands	0.2910	0.3933	0.4834
Burkina Faso	0.1743	0.2644	0.3501	Sri Lanka	0.3249	0.3967	0.4592
Rwanda	0.1967	0.2645	0.3318	Philippines	0.3070	0.4123	0.5107
Thailand	0.1146	0.2731	0.4223	United Kingdom	0.3066	0.4181	0.5140
Belgium	0.1702	0.2824	0.3870	Hong Kong	0.3203	0.4198	0.5054
Greece	0.1729	0.2875	0.3854	Congo, Dem. Rep.	0.3585	0.4428	0.5217
Gabon	0.1131	0.2876	0.4530	Greece	0.3183	0.4545	0.5778
Ireland	0.0243	0.2973	0.6343	Italy	0.3929	0.4579	0.5186
Papua New Guinea	0.2084	0.3019	0.3906	Finland	0.3202	0.4767	0.6167
Suriname	0.2593	0.3330	0.3902	Suriname	0.3780	0.4806	0.5636
Congo, Dem. Rep.	0.2255	0.3564	0.4735	Sweden	0.3855	0.4980	0.5978
Finland	0.1070	0.4032	0.6541	Pakistan	0.4043	0.5233	0.6102
Brazil	0.1794	0.4140	0.6180	Singapore	0.4548	0.5249	0.5875
Togo	0.3029	0.4272	0.5388	Belgium	0.3331	0.5626	0.7618

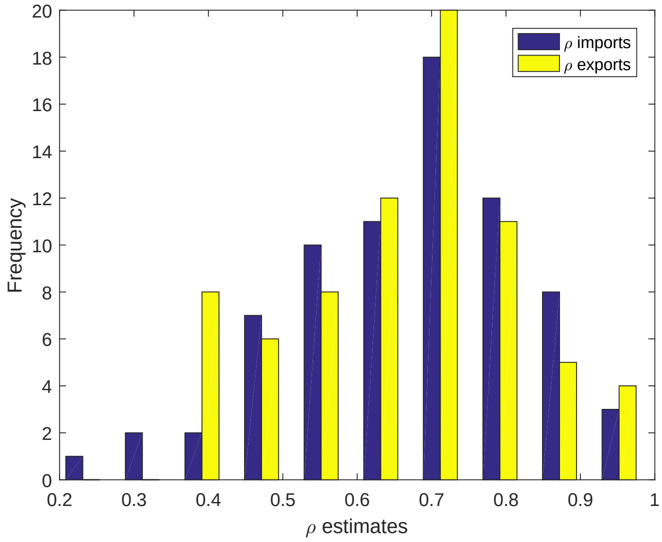

Fig. 1: Distribution of estimates for the 74 scalar dependence parameters ρ from SAR model estimates

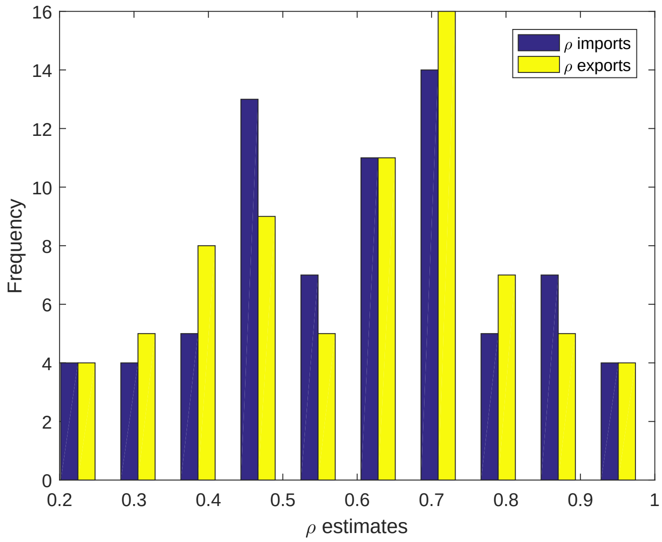

Fig. 2: Distribution of estimates for the 74 scalar dependence parameters ρ from SDM model estimates

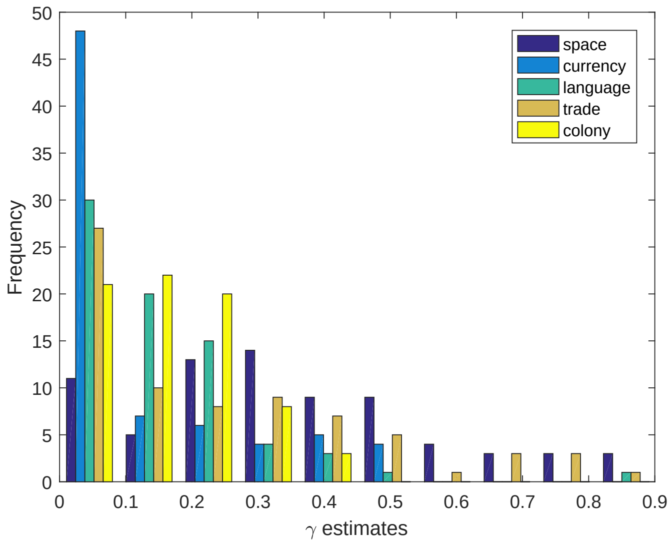

Fig. 3: Distribution of estimates for the 74 γ parameters from the SDM import model estimates

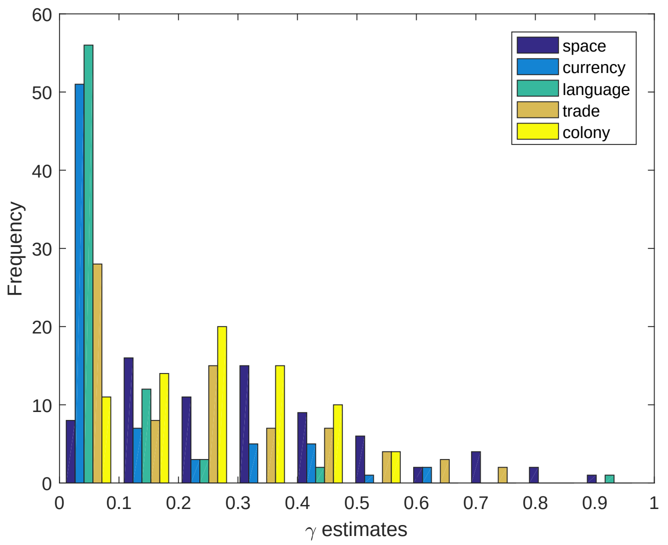

Fig. 4: Distribution of estimates for the 74 γ parameters from the SDM export model estimates

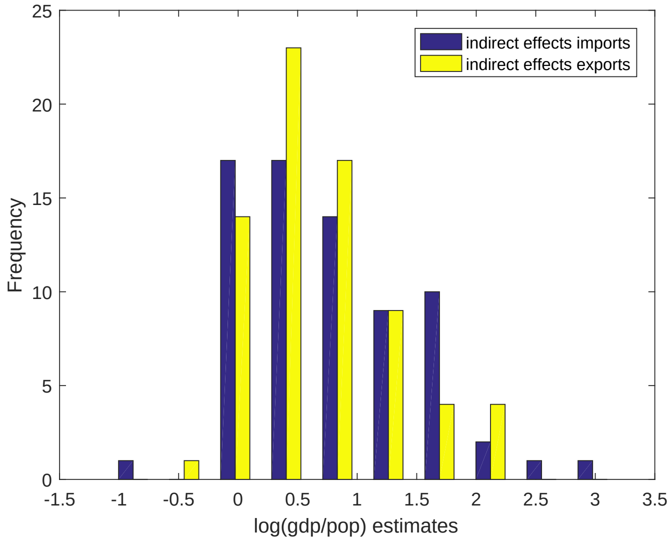

Fig. 5: Frequency distribution of the posterior mean indirect effects from SAR models of imports and exports

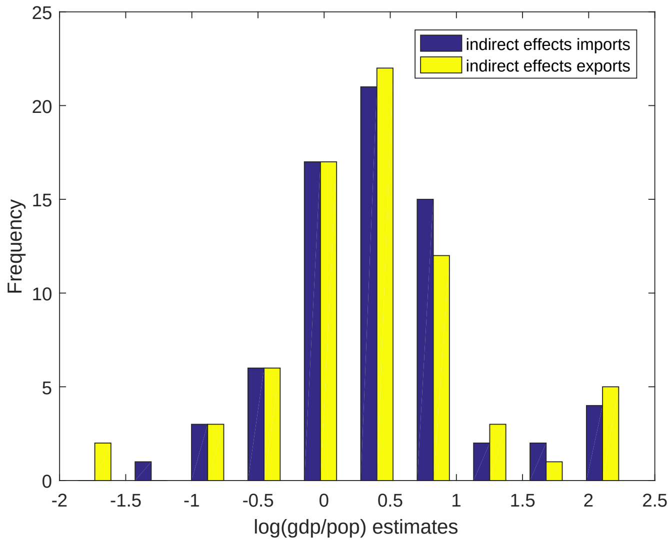

Fig. 6: Frequency distribution of the posterior mean indirect effects from SDM models of imports and exports